ОТВЕТЫ К ДОМАШНЕМУ ЗАДАНИЮ

«ФУНКЦИОНАЛЬНЫЕ РЯДЫ»

Вариант 1.

1. $\sum_{n=2}^{\infty}(-1)^n\cdot\frac{(x+7)^n\ln n}{n\cdot 3^n}$

$\lim_{n\to\infty}\frac{|x+7|^{n+1}\cdot\ln(n+1)\cdot n\cdot 3^n}{3^{n+1}\cdot(n+1)|x+7|^n\cdot\ln n}=\lim_{n\to\infty}\frac{|x+7|^n\cdot n\cdot\ln(n+1)}{3\cdot(n+1)\cdot\ln n}=\frac{|x+7|}{3}$

$|x+7|<3$

$\underline{x+7=3}$

$\sum_{n=2}^{\infty}(-1)^n\cdot\frac{\ln n}{n}$; $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ - расходится

$\sum_{n=2}^{\infty}(-1)^n\cdot\frac{\ln n}{n}$ - сходится условно (по признаку Лейбница).

$\underline{x+7=-3}$

$\sum_{n=2}^{\infty}(-1)^n\cdot\frac{(-3)^n\cdot\ln n}{n}=\sum_{n=2}^{\infty}\frac{\ln n}{n}$ - расходится

<u>Ответ:</u> $-10<x\le -4$

2. $f(x)=xe^{-x}$

$xe^{-x}=\left[(x-1)+1\right]\cdot e^{-(x-1)}\cdot e^{-1}$

$e^{-(x-1)}=1-(x-1)+\frac{(x-1)^2}{2!}-\dots+(-1)^n\cdot\frac{(x-1)^n}{n!}+\dots$

$-\infty<x<+\infty$

$(x-1)\cdot e^{-(x-1)}=(x-1)-(x-1)^2+\dots+(-1)^{n-1}\cdot\frac{(x-1)^n}{(n-1)!}+\dots$

<u>Ответ:</u> $xe^{-x}=e^{-1}+e^{-1}\cdot\sum_{n=2}^{\infty}(-1)^{n-1}\cdot\frac{1}{(n-2)!\cdot n}\cdot(x-1)^n$

$-\infty<x<+\infty$

3. $\int_0^2\frac{1}{x}\cdot sh\frac{x^2}{4}dx$

$sh\frac{x^2}{4}=\frac{x^2}{4}+\frac{x^6}{3!\cdot 4^3}+\dots+\frac{x^{2(2n-1)}}{(2n-1)!\cdot 4^{2n-1}}+\dots$

$-\infty<x<+\infty$

$\frac{1}{x}\cdot sh\frac{x^2}{4}=\frac{x}{4}+\frac{x^5}{3!\cdot 4^3}+\frac{x^9}{5!\cdot 4^5}+\dots+\frac{x^{4n-3}}{(2n-1)!\cdot 4^{2n-1}}+\dots$

$-\infty<x<+\infty$

При $x=0$ $S(x)=0$ (где $S(x)$ - сумма ряда)

$$\int_0^x \frac{1}{x}\, sh\frac{x^2}{4}\,dx = \frac{x^2}{4\cdot 2} + \frac{x^6}{3!\,4^3\cdot 6} + \frac{x^{10}}{5!\,4^5\cdot 10} + \dots + \frac{x^{4n-2}}{(2n-1)!\,4^{2n-1}\cdot(4n-2)} + \cdots$$

$-\infty < x < +\infty$

$$\int_0^2 \frac{1}{x}\, sh\frac{x^2}{4}\,dx = \frac{2^2}{4\cdot 2} + \frac{2^6}{3!\,4^3\cdot 6} + \frac{2^{10}}{5!\,4^5\cdot 10} + \dots + \frac{2^{4n-2}}{(2n-1)!\,4^{2n-1}\cdot(4n-2)} + \dots =$$

$$= \frac{1}{2} + \frac{1}{3!\,6} + \frac{1}{5!\,10} + \dots + \frac{1}{(2n-1)!\,(4n-2)} + \cdots$$

$$R_n = \frac{1}{(2n+1)!\,(4n+2)} + \frac{1}{(2n+3)!\,(4n+6)} + \dots < \frac{1}{(2n+1)!\,(4n+2)} \times$$

$$\times\left(1 + \frac{1}{(2n+2)^2} + \frac{1}{(2n+2)^4} + \dots\right) = \frac{1}{(2\text{[illegible]}n+2)} \cdot \frac{1}{1-\left(\frac{1}{2n+2}\right)^2} =$$

$$= \frac{(2n+2)^2}{(2n+1)!\,(4n+2)\cdot[(2n+2)^2-1]}$$

[illegible]

[illegible] $\frac{64}{10^{?}\cdot 12\cdot 65} < 0{,}00002$

$$\int_0^2 \frac{1}{x}\cdot sh\frac{x^2}{4}\,dx \approx \frac{1}{2} + \frac{1}{3!\,6} + \frac{1}{5!\,10}$$

$\frac{1}{2} = 0{,}5$

$\frac{1}{3!\,6} = 0{,}02$[illegible] [illegible] с избытком

$\frac{1}{5!\,10} = 0{,}00$[illegible] [illegible] с избытком

0,529

$$0{,}528 < \int_0^2 \frac{1}{x}\cdot sh\frac{x^2}{4}\,dx < 0{,}529$$

Ответ: $\int_0^2 \frac{1}{x}\cdot sh\frac{x^2}{4}\,dx = 0{,}529$

4. $y' = e^y - xy$ $\qquad y|_{x=3} = 0$

$$y(x) = y(3) + y'(3)(x-3) + \frac{y''(3)}{2!}\cdot(x-3)^2 + \frac{y'''(3)}{3!}\cdot(x-3)^3 + \cdots$$

$y(3) = 0$

$y'(3) = 1$

$y''(3) = 1 - 3 = -2$ $\qquad y'' = e^y\cdot y' - y - y'x$

$y'''(3) = 1 - 2 - 2 + 6 = 3$ $\qquad y''' = e^y\cdot(y')^2 + e^y\cdot y'' - 2y' - y''x$

Ответ: $y(x) = (x-3) - (x-3)^2 + \frac{1}{2!}\cdot(x-3)^3 + \cdots$

5. $(x^2+1)y''+4xy'+2y=2x \qquad y(0)=\frac{1}{3}, \qquad y'(0)=0$

$$y(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+\cdots$$

$$y'(x)=a_1+2a_2x+3a_3x^2+4a_4x^3+\cdots+(n+1)a_{n+1}x^n+\cdots$$

$$y''(x)=2a_2+3\cdot 2a_3x+4\cdot 3a_4x^2+\cdots+(n+2)(n+1)a_{n+2}x^n+\cdots$$

$$2y=2a_0+2a_1x+2a_2x^2+2a_3x^3+\cdots+2a_nx^n+\cdots$$

$$4xy'=4a_1x+2\cdot 4a_2x^2+3\cdot 4a_3x^3+\cdots+4na_nx^n+\cdots$$

$$y''=2a_2+3\cdot 2a_3x+4\cdot 3a_4x^2+\cdots+(n+2)(n+1)a_{n+2}x^n+\cdots$$

$$x^2y''=2a_2x^2+\cdots+n(n-1)a_nx^n+\cdots$$

$$2a_n+4na_n+n(n-1)a_n+(n+2)(n+1)a_{n+2}=0$$

$n\neq 1 \qquad a_n(n+2)(n+1)+(n+2)(n+1)a_{n+2}=0$

$$a_{n+2}=-a_n$$

$n=1 \qquad 6a_1+6a_3=2 \qquad a_3=\frac{2-6a_1}{6}$

$a_0=\frac{1}{3} \qquad a_2=-\frac{1}{3};\cdots; \qquad a_{2n}=(-1)^n\cdot\frac{1}{3} \qquad n=0,1,\ldots$

$a_1=0 \qquad a_3=\frac{1}{3};\cdots; \qquad a_{2n-1}=(-1)^n\cdot\frac{1}{3} \qquad n=1,2,\ldots$

$$y(x)=\frac{1}{3}-\frac{1}{3}x^2+\frac{1}{3}x^3+\frac{1}{3}x^4-\cdots$$

$$y\left(\frac{1}{2}\right)=\frac{1}{3}+\left(-\frac{1}{3}\right)\left(\frac{1}{2}\right)^2+\frac{1}{3\cdot 2^3}+\frac{1}{3\cdot 2^4}-\cdots$$

$$|R_n|<\frac{1}{3\cdot 2^{n+1}}+\frac{1}{3\cdot 2^{n+2}}+\cdots=\frac{1}{3\cdot\left(1-\frac{1}{2}\right)\cdot 2^{n+1}}=\frac{1}{3\cdot 2^{n+2}}$$

$n=7 \qquad |R_7|<\frac{1}{3\cdot 2^9}<0{,}005$

	$\frac{1}{3}=0{,}333$	(+)
-	$\frac{1}{3}\cdot\left(\frac{1}{2}\right)^2=0{,}083$	(+)
+	$\frac{1}{3}\cdot\left(\frac{1}{2}\right)^3=0{,}042$	(-)
+	$\frac{1}{3}\cdot\left(\frac{1}{2}\right)^4=0{,}021$	(+)
-	$\frac{1}{3}\cdot\left(\frac{1}{2}\right)^5=0{,}010$	(+)
-	$\frac{1}{3}\cdot\left(\frac{1}{2}\right)^6=0{,}005$	(+)
	0,301	

$$0{,}301-0{,}003<y\left(\frac{1}{2}\right)<0{,}301+0{,}002$$

__Ответ:__ $y\left(\frac{1}{2}\right)=0{,}30$

6. $f(x)=\begin{cases}1 & (-\pi,0)\\ x & (0,\pi)\end{cases}$

$$a_0=\frac{1}{\pi}\cdot\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\cdot\left(\int_{-\pi}^{0}dx+\int_{0}^{\pi}xdx\right)=\frac{1}{\pi}\cdot\left(\pi+\frac{x^2}{2}\bigg|_0^{\pi}\right)=\frac{2+\pi}{2}$$

$$a_n=\frac{1}{\pi}\cdot\left(\int_{-\pi}^{0}\cos nxdx+\int_{0}^{\pi}x\cos nxdx\right)=\frac{1}{\pi}\cdot\left(\frac{\sin nx}{n}\bigg|_{-\pi}^{0}+\frac{x\sin nx}{n}\bigg|_0^{\pi}+\frac{\cos nx}{n^2}\bigg|_0^{\pi}\right)=\frac{(-1)^n-1}{\pi\cdot n^2}$$

$$b_n=\frac{1}{\pi}\cdot\left(\int_{-\pi}^{0}\sin nxdx+\int_{0}^{\pi}x\sin nxdx\right)=\frac{1}{\pi}\cdot\left(-\frac{\cos nx}{n}\bigg|_{-\pi}^{0}-\frac{x\cos nx}{n}\bigg|_0^{\pi}+\frac{\sin nx}{n^2}\bigg|_0^{\pi}\right)=$$

$$=\frac{1}{\pi}\cdot\left[\frac{(-1)^n-1}{n}-\frac{\pi\cdot(-1)^n}{n}\right]$$

__Ответ:__ $f(x)=\frac{2+\pi}{4}+\frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cdot\cos nx+\frac{(-1)^n-1-\pi\cdot(-1)^n}{n}\cdot\sin nx$

7. $f(x)=4-x$ $\quad 0<x<4$ по косинусам

$$a_0=\frac{1}{2}\cdot\int_0^4(4-x)dx=\frac{1}{2}\cdot\left(4x-\frac{x^2}{2}\right)\bigg|_0^4=4$$

$$a_n=\frac{1}{2}\cdot\int_0^4(4-x)\cos\frac{\pi nx}{4}dx=\frac{1}{2}\cdot\left[(4-x)\sin\frac{\pi nx}{4}\cdot\frac{4}{\pi n}\bigg|_0^4+\frac{4}{\pi n}\cdot\int_0^4\sin\frac{\pi nx}{4}dx\right]=$$

$$=-\frac{2}{\pi n}\cos\frac{\pi nx}{4}\cdot\frac{4}{\pi n}\bigg|_0^4=-\frac{8}{\pi^2n^2}\cdot\left[(-1)^2-1\right]=\begin{cases}0 & n=2m\\ \dfrac{16}{\pi^2(2m-1)^2} & n=2m-1\end{cases}$$

__Ответ:__ $4-x=2+\sum_{m=1}^{\infty}\frac{16}{\pi^2(2m-1)^2}\cdot\cos\frac{(2m-1)\pi x}{4}$

Вариант 2.

1. $$\sum_{n=1}^{\infty}\frac{(x+2)^n}{2^{2n}\cdot\left(\sqrt[4]{n}-tg\frac{1}{n}\right)}$$

$$\lim_{n\to\infty}\frac{|x+2|^{n+1}\cdot 2^{2n}\cdot\left(\sqrt[4]{n}-tg\frac{1}{n}\right)}{2^{2(n+1)}\cdot\left(\sqrt[4]{n+1}-tg\frac{1}{n+1}\right)\cdot|x+2|^n}=\frac{|x+2|}{2^2}$$

$|x+2|<4$

$\underline{x+2=4}$

$$\sum_{n=1}^{\infty}\frac{4^n}{2^{2n}\cdot\left(\sqrt[4]{n}-tg\frac{1}{n}\right)}=\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}-tg\frac{1}{n}}$$

$$\frac{1}{\sqrt[4]{n}-tg\frac{1}{n}}\sim\frac{1}{\sqrt[4]{n}}\qquad(n\to\infty)$$ ряд расходится

$\underline{x+2=-4}$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}-tg\frac{1}{n}}$$ ряд сходится условно,

так как $$\frac{\sqrt[4]{n+1}-tg\frac{1}{n+1}}{\sqrt[4]{n}-tg\frac{1}{n}}=\sqrt[4]{\frac{n+1}{n}}\cdot\frac{1-\frac{1}{\sqrt[4]{n+1}}\cdot tg\frac{1}{n+1}}{1-\frac{1}{\sqrt[4]{n}}\cdot tg\frac{1}{n}}>1$$

$$\frac{1}{\sqrt[4]{n}-tg\frac{1}{n}}\to 0\qquad(n\to\infty)$$

Ответ: $-6\le x<2$

2. xe^{-3x}; $\quad xe^{-3x}=\left[(x-3)+3\right]\cdot e^{-3(x-3)}\cdot e^{-9}$

$$3e^{-3(x-3)}=3-3^2(x-3)+\frac{3^3}{2!}\cdot(x-3)^2+\dots+(-1)^n\cdot\frac{3^{n+1}\cdot(x-3)^n}{n!}+\dots$$

$$(x-3)e^{-3(x-3)}=(x-3)-3(x-3)^2+\dots+(-1)^{n-1}\frac{3^{n-1}\cdot(x-3)^n}{(n-1)!}+\dots$$

$$xe^{-3x}=3e^{-9}+e^{-9}\cdot\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 3^{n-1}\cdot(3^2-n)}{n!}\cdot(x-3)^n$$

$$-\infty<x<+\infty$$

Ответ: $$xe^{-3x}=3e^{-9}+e^{-9}\cdot\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 3^{n-1}\cdot(3^2-n)}{n!}\cdot(x-3)^n$$

3. $$\int_0^{0.5}\frac{\sin 3x}{x}dx$$

$$\frac{\sin 3x}{x} = 3 - 3^3 \cdot \frac{x^2}{3!} + \frac{3^5 \cdot x^4}{5!} - \dots + (-1)^{n-1} \cdot \frac{3^{2n-1} \cdot x^{2 [illegible]}}{(2n-1 [illegible]}$$

$-\infty < x < +\infty \qquad S(x)\big|_{x=0} = 3!$

$$\int_0^x \frac{\sin 3x}{x} dx = 3x - \frac{3^3 \cdot x^3}{3 \cdot 3!} + \frac{3^5 \cdot x^5}{5 \cdot 5!} - \dots + (-1)^{n-1} \cdot \frac{3^{[illegible]}}{(2n - [illegible]}$$

$-\infty < x < +\infty$

$$\int_0^{0,5} \frac{\sin 3x}{x} dx = 3 \cdot 0{,}5 - \frac{3^3 \cdot (0{,}5)^3}{3 \cdot 3!} + \frac{3^5 \cdot (0{,}5)^5}{5 \cdot 5!} - \dots + (- [illegible] \frac{[illegible] \cdot (0{,}5)^{2n-1}}{[illegible] n-1)(2n-1)!} + \dots$$

$$|R_3| = \frac{3^7 \cdot (0{,}5)^7}{7 \cdot 7!} < 0{,}0005$$

$+ \quad 3 \cdot 0{,}5 = 1{,}5$

$- \quad \frac{3 \cdot (0{,}5)^3}{2} = 0{,}188 \quad (-)$

$+ \quad \frac{3^5 \cdot (0{,}5)^5}{5 \cdot 5!} = 0{,}013 \quad (-)$

4,325

$$1{,}3245 < \int_0^{0,5} \frac{\sin 3x}{x} dx < 1{,}3260$$

Ответ: $\int_0^{0,5} \frac{\sin 3x}{x} dx = 1{,}325$

4. $y' = 2x - \cos y \qquad y\big|_{x=1} = \frac{\pi}{2}$

$y(1) = \frac{\pi}{2}$

$y'(1) = 2$

$y''(1) = 2 + 2 = 4 \qquad y'' = 2 + \sin y \cdot y'$

$y'''(1) = 4 \qquad y''' = y'' \sin y + (y')^2 \cdot \cos y$

$$y(x) = \frac{\pi}{2} + 2(x-1) + \frac{4(x-1)^2}{2!} + \frac{4(x-1)^3}{3!} + \dots$$

Ответ: $y(x) = \frac{\pi}{2} + 2(x-1) + \frac{4(x-1)^2}{2!} + \frac{4(x-1)^3}{3!} + \dots$

5. $(x^2 - 3)y'' - 2xy' + 2y = (x^2 - 1) \cdot chx - 2x \cdot shx$

$y(0) = 1 \qquad y'(0) = 2$

$- \quad chx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + \dots$

$+ \quad x^2 chx = x^2 + \frac{x^4}{2!} + \dots + \frac{x^{2n}}{(2n-2)!} + \dots$

$- \quad 2x \cdot shx = 2x^2 + \frac{2x^4}{3!} + \dots + \frac{2x^{2n}}{(2n-1)!} + \dots$

$$(x^2-1)\cdot chx - 2x\cdot shx = 1+\sum_{n=1}^{\infty}\frac{4n^2-6n-1}{(2n)!}\cdot x^{2n}$$

$$\begin{array}{r|l}
2 & y = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n + \cdots \\
-2 & xy' = \quad a_1x + 2a_2x^2 + 3a_3x^3 + \cdots + na_nx^n + \cdots \\
-3 & y'' = \quad 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \cdots (n+2)(n+1)a_{n+2}\cdot x^n + \cdots \\
1 & x^2y'' = \quad 2a_2x^2 + 3\cdot 2a_3x^3 + \cdots + n(n-1)a_nx^n + \cdots \\
\hline
\end{array}$$

$$(x^2-3)y'' - 2xy' + 2y = (-6a_2 + 2a_0) + (-3)\cdot 2\cdot 3a_3x + (-3)\cdot 4\cdot 3a_4x^2 +$$

$$+[(-3)\cdot 54a_5 + 2a_3]x^3 + \cdots + [(-3)(n+2)(n+1)a_{n+2} + (n-1)(n-2)a_n]x^n + \cdots$$

$a_0 = 1$ $\qquad$ $a_1 = 2$

$2a_0 - 6a_2 = 1$ $\qquad$ $6a_2 = 1$ $\qquad$ $a_2 = \frac{1}{3!}$

$-3\cdot 2\cdot 3a_3 = 0$ $\qquad$ $a_3 = 0$

$n \ge 3$

$(-3)(n+2)(n+1)a_{n+2} + (n-1)(n-2)a_n = 0$ при нечетном n, следовательно, $a_{2n-1} = 0$

Рассмотрим коэффициенты при четных степенях.

$$(-3)(2n+2)(2n+1)a_{2n+2} + (2n-1)(2n-2)a_{2n} = \frac{4n^2-6n-1}{(2n)!}$$

$(-3)\cdot 4\cdot 3a_4 = \frac{4-6-1}{2}$ $\qquad$ $a_4 = \frac{1}{4!}$

$(-3)\cdot 6\cdot 5a_6 \neq 3\cdot 2a_4 = \frac{4\cdot 4-12-1}{4!};$ $\qquad$ $6\cdot 5a_6 = \frac{2}{4!} - \frac{1}{4!}$ $\qquad$ $a_6 = \frac{1}{6!}$

$$a_{2n} = \frac{1}{(2n)!}$$

$$(-3)(2n+2)(2n+1)a_{2n+2} + (2n-1)(2n-2)\cdot\frac{1}{(2n)!} = \frac{4n^2-6n-1}{(2n)!}$$

$$(-3)(2n+2)(2n+1)a_{2n+2} = \frac{4n^2-6n-1-4n^2+6n-2}{(2n)!}$$

$$a_{2n+2} = \frac{1}{(2n+2)!}$$

$$y(x) = 1 + 2x + \frac{1}{3}\cdot x^2 + \frac{1}{4!}\cdot x^4 + \frac{1}{6!}\cdot x^6 + \cdots + \frac{1}{(2n)!}\cdot x^{2n} + \cdots = 2x + chx$$

$$y(1) = 1 + 2 + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{6!} + \cdots + \frac{1}{(2n)!} + \cdots$$

$$\frac{1}{(2n+2)!} + \frac{1}{(2n+4)!} + \cdots < \frac{1}{(2n+2)!}\cdot\left[1 + \frac{1}{(2n+3)^2} + \frac{1}{(2n+3)^4} + \cdots\right] =$$

$$= \frac{1}{(2n+2)!}\cdot\frac{1}{1-\frac{1}{(2n+3)^2}} = \frac{(2n+3)^2}{(2n+2)!\cdot[(2n+3)^2-1]}$$

$\underline{n=2}$ $\qquad$ $R < 0{,}005$

$y(1) \approx 1 + 2 + \frac{1}{3!} + \frac{1}{4!}$

$\frac{1}{3!} = 0{,}333 \quad (+)$

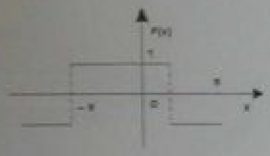
$\frac{1}{4!} = 0{,}042 \quad (-)$

$\overline{\quad\quad\quad\quad\quad\quad}$
$0{,}375$

$3{,}375 - 0{,}005 < y(1) < 3{,}375 + 0{,}010$

$3{,}370 < y(1) < 3{,}385$

Ответ: $y(1) \approx 3{,}38$



6. $f(x) = \begin{cases} 1 & -\pi < x < \frac{\pi}{3} \\ -1 & \frac{\pi}{3} < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{\pi/3} dx + \int_{\pi/3}^{\pi} (-1) dx \right) = \frac{1}{\pi} \cdot \int_{-\pi/3}^{\pi/3} dx = \frac{2}{3}$$

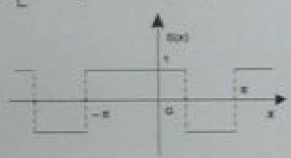
$$a_n = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{\pi/3} \cos nx dx + \int_{\pi/3}^{\pi} -\cos nx dx \right) = \frac{2}{\pi} \cdot \int_{0}^{\pi/3} \cos nx dx = \frac{2}{\pi} \cdot \frac{\sin nx}{n} \bigg|_0^{\pi/3} = \frac{2 \sin \frac{\pi n}{3}}{\pi n}$$

$$b_n = \frac{1}{\pi} \cdot \left(\int_{-\pi}^{\pi/3} \sin nx dx + \int_{\pi/3}^{\pi} -\sin nx dx \right) = \frac{2}{\pi} \cdot \int_{\pi/3}^{\pi} -\sin nx dx = \frac{2}{\pi} \cdot \frac{\cos nx}{n} \bigg|_{\pi/3}^{\pi} =$$

$$= \frac{2}{\pi n} \cdot \left[(-1)^n - \cos \frac{\pi n}{3} \right]$$

$$f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \left[\sin \frac{\pi n}{3} \cos nx + \left((-1)^n - \cos \frac{\pi n}{3} \right) \cdot \sin nx \right]$$

Ответ: $f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \left[\sin \frac{\pi n}{3} \cdot \cos nx + \left((-1)^n - \cos \frac{\pi n}{3} \right) \cdot \sin nx \right]$



7. $f(x) = \begin{cases} -1 & 0 < x < 1 \\ -x^2 & 1 < x < 2 \end{cases}$ по косинусам

$$a_0=\frac{2}{2}\cdot\left(\int_0^1-dx+\int_1^2-x^2dx\right)=\left(-x\Big|_0^1-\frac{x^3}{3}\Big|_1^2\right)=-1-\frac{8}{3}+\frac{1}{3}=-\frac{10}{3}$$

$$a_n=\frac{2}{2}\cdot\left(\int_0^1-\cos\frac{\pi nx}{2}dx-\int_1^2 x^2\cos\frac{\pi nx}{2}dx\right)=-\frac{2}{\pi n}\cdot\sin\frac{\pi nx}{2}\Big|_0^1-\left(x^2\cdot\frac{2}{\pi n}\cdot\sin\frac{\pi nx}{2}\Big|_1^2-\right.$$

$$\left.-\frac{4}{\pi n}\int_1^2 x\sin\frac{\pi nx}{2}dx\right)=-\frac{2}{\pi n}\sin\frac{\pi n}{2}+\frac{2}{\pi n}\sin\frac{\pi n}{2}+\frac{4}{\pi n}\cdot\left(-x\cdot\frac{2}{\pi n}\cos\frac{\pi nx}{2}\Big|_1^2+\right.$$

$$\left.+\frac{8}{\pi^2 n^2}\cdot\frac{2}{\pi n}\cdot\sin\frac{\pi n}{2}\Big|_1^2\right)=-\frac{8}{\pi n^2}\cdot\left[2\cos\pi n-\cos\frac{\pi n}{2}\right]-\frac{16}{\pi^3 n^3}\cdot\sin\frac{\pi n}{2}=$$

$$=-\frac{8}{\pi n^2}\cdot\left[2\cdot(-1)^n-\cos\frac{\pi n}{2}\right]-\frac{16}{\pi^3 n^3}\cdot\sin\frac{\pi n}{2}$$

__**Ответ:**__ $f(x)=-\frac{5}{3}+\sum_{n=1}^{\infty}\left[-\frac{8}{\pi n^2}\cdot\left(2\cdot(-1)^n-\cos\frac{\pi n}{2}\right)-\frac{16}{\pi^3 n^3}\cdot\sin\frac{\pi n}{2}\right]\cdot\cos\frac{\pi nx}{2}$

Вариант 3.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{3^{2n}\cdot\left(x-\frac{1}{9}\right)^n}{2^{3n}}\cdot\sin\frac{1}{n}$

$$\lim_{n\to\infty}\frac{3^{2n+2}\cdot\left|x-\frac{1}{9}\right|^{n+1}\cdot\sin\frac{1}{n+1}\cdot 2^{3n}}{2^{3n+3}\cdot 3^{2n}\cdot\left|x-\frac{1}{9}\right|^n\cdot\sin\frac{1}{n}}=\frac{3^2}{2^3}\cdot\left|x-\frac{1}{9}\right|$$

$\left|x-\frac{1}{9}\right|<\frac{8}{9}$

$x-\frac{1}{9}=-\frac{8}{9}$

$\sum_{n=1}^{\infty}\frac{3^{2n}\cdot 2^{3n}}{2^{3n}\cdot 3^{2n}}\cdot\sin\frac{1}{n}=\sum_{n=1}^{\infty}\sin\frac{1}{n};$ - ряд расходится, так как $\sin\frac{1}{n}\sim\frac{1}{n}$ $(n\to\infty)$

$x-\frac{1}{9}=\frac{8}{9}$

$\sum_{n=1}^{\infty}(-1)^n\cdot\sin\frac{1}{n}$ - сходится условно (признак Лейбница).

__**Ответ:**__ $-\frac{7}{9}<x\le 1$

2. $(x+3)\cdot e^x$

$(x+3)\cdot e^x=\left[(x+2)+1\right]\cdot e^{x+2}\cdot e^{-2}$

$e^{x+2}=1+(x+2)+\frac{1}{2!}\cdot(x+2)^2+\dots+\frac{1}{n!}\cdot(x+2)^n+\cdots$

$$(x+2)\cdot e^{x+2}=(x+2)+(x+2)^2+\dots+\frac{1}{(n-1)!}\cdot(x+2)^n+\dots$$

$$-\infty<x<+\infty$$

$$(x+3)\cdot e^x=e^{-2}\cdot\sum_{n=0}^{\infty}\frac{n+1}{n!}\cdot(x+2)^n$$

$$-\infty<x<+\infty$$

Ответ: $(x+3)\cdot e^x=e^{-2}\cdot\sum_{n=0}^{\infty}\frac{n+1}{\cdot n!}\cdot(x+2)^n$

$$-\infty<x<+\infty$$

3. $\int_0^{1,5}x^4\cdot\cos\frac{x}{5}dx$

$$x^4\cdot\cos\frac{x}{5}=x^4-\frac{x^6}{2!\,5^2}+\frac{x^8}{4!\,5^4}-\dots+(-1)^n\cdot\frac{x^{2n+4}}{(2n)!\,5^{2n}}+\dots$$

$$\int_0^x x^4\cdot\cos\frac{x}{5}dx=\frac{x^5}{5}-\frac{x^7}{2!\,5^2\cdot7}+\frac{x^9}{4!\,5^4\cdot9}-\dots+(-1)^n\cdot\frac{x^{2n+5}}{(2n)!\,5^{2n}\cdot(2n+5)}+\dots$$

$$-\infty<x<+\infty$$

$$\int_0^{1,5}x^4\cdot\cos\frac{x}{5}dx=\frac{(1,5)^5}{5}-\frac{(1,5)^7}{2!\,5^2\cdot7}+\frac{(1,5)^9}{4!\,5^4\cdot9}-\dots+(-1)^n\cdot\frac{(1,5)^{2n+5}}{(2n)!\,5^{2n}\cdot(2n+5)}+\dots$$

$$|R_n|<\frac{(1,5)^{2n+5}}{(2n)!\,5^{2n}\cdot(2n+5)}$$

R_n - остаток ряда после первых n- членов

$n=2$

$$|R_2|<\frac{(1,5)^9}{4!\,5^4\cdot9}<0,0003$$

+	$\frac{(1,5)^5}{5}=1,5188$	(-)	с избытком
-	$\frac{(1,5)^7}{2!\,5^2\cdot7}=0,0488$	(+)	с недостатком
	1,4700		

$$1,469<\int_0^{1,5}x^4\cdot\cos\frac{x}{5}dx<1,471$$

Ответ: $\int_0^{1,5}x^4\cdot\cos\frac{x}{5}dx\approx1,470$

4. $y'=x^2+y^3$ $\qquad y|_{x=1}=1$

$$y(x)=y(1)+\frac{y'(1)}{1!}\cdot(x-1)+\frac{y''(1)}{2!}\cdot(x-1)^2+\frac{y'''(1)}{3!}\cdot(x-1)^3+\dots$$

$y(1)=1$

$y'(1)=2$ $\qquad y'=x^2+y^3$

$y''(1)=2+6=8$ $\qquad y''=2x+3y^2\cdot y'$

$y'''(1) = 2 + 24 + 24 = 50 \qquad\qquad y''' = 2 + 6y\cdot(y')^2 + 3y^2y''$

$$y(x) = 1 + 2(x-1) + \frac{8}{2!}(x-1)^2 + \frac{50}{3!}(x-1)^3 + \ldots$$

<u>**Ответ:**</u> $\quad y(x) = 1 + 2(x-1) + \frac{8}{2!}(x-1)^2 + \frac{50}{3!}(x-1)^3 + \ldots$

5. $\quad (x-1)^2 y'' - 2(x-1)y' + 2y = (x^2+1)\cdot e^{-x}$

$y(0) = 1 \qquad\qquad y'(0) = -1$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + (-1)^n\cdot\frac{x^n}{n!} + \ldots$$

$$x^2e^{-x} = x^2 - x^3 + \ldots + (-1)^n\cdot\frac{x^n}{(n-2)!} + \ldots$$

$$(x^2+1)\cdot e^{-x} = \sum_{n=0}^{\infty}(-1)^n\cdot\frac{n^2-n+1}{n!}\cdot x^n$$

$$\begin{array}{r|l}
2 & y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n + \cdots \\
2 & y'(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots + (n+1)a_{n+1}x^n + \cdots \\
-2 & xy'(x) = a_1x + 2a_2x^2 + 3a_3x^3 + \cdots + na_n\cdot x^n + \cdots \\
1 & y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots \\
-2 & xy''(x) = 2a_2x + 3\cdot 2a_3x^2 + 4\cdot 3a_4x^3 + \cdots + (n+1)\cdot n\cdot a_{n+1}x^n + \cdots \\
1 & x^2y''(x) = 2a_2x^2 + 3\cdot 2a_3x^3 + \cdots + n(n-1)a_nx^n + \cdots \\
\hline
\end{array}$$

$$(x-1)^2y'' - 2(x-1)y' + 2y = \sum_{n=0}^{\infty}\left[(n+2)(n+1)a_{n+2} - 2(n^2-1)a_{n+1} + (n-1)(n-2)a_n\right]\cdot x^n$$

$a_0 = 1 \qquad\qquad a_1 = -1$

$2a_2 + 2 - 2 = 1 \qquad\qquad a_2 = \frac{1}{2}$

$3\cdot 2a_3 - 0\cdot a_2 - 0\cdot a_1 = -1 \qquad\qquad a_3 = -\frac{1}{3\cdot 2}$

$4\cdot 3a_4 + 2\cdot 3\cdot\frac{1}{3\cdot 2} = \frac{3}{2} \qquad\qquad a_4 = \frac{1}{2\cdot 3\cdot 4}$

....

$$a_n = (-1)^n\cdot\frac{1}{n!}$$

$$a_{n+1} = (-1)^{n+1}\cdot\frac{1}{(n+1)!}$$

$$(n+2)\cdot(n+1)\cdot a_{n+2} - 2\cdot(n^2-1)\cdot\frac{(-1)^{n+1}}{(n+1)!} + (n-1)\cdot(n-2)\cdot\frac{(-1)^n}{n!} = (-1)^n\cdot\frac{n^2-n+1}{n!}$$

$$a_{n+2} = \frac{1}{(n+2)(n+1)}\cdot\left[\frac{(-1)^{n+1}\cdot 2(n-1) - (-1)^n\cdot(n-1)(n-2) + (-1)^n\cdot(n^2-n+1)}{n!}\right] =$$

$$= \frac{(-1)^n}{(n+2)!}\cdot\left[-2\cdot(n-1) - (n-1)(n-2) + n^2 - n + 1\right] = \frac{(-1)^n}{(n+2)!} = \frac{(-1)^{n+2}}{(n+2)!}$$

$$y(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\cdot x^n \qquad\qquad y(-1)=\sum_{n=0}^{\infty}\frac{1}{n!}$$

$$R_n<\frac{1}{n!}+\frac{1}{(n+1)!}+\ldots=\frac{1}{n!}\left(1+\frac{1}{n+1}+\frac{1}{(n+1)(n+2)}+\ldots\right)<\frac{1}{n!}\cdot\left(1+\frac{1}{n+1}+\frac{1}{(n+1)^2}+\ldots\right)=$$

$$=\frac{1}{n!}\cdot\frac{1}{1-\frac{1}{n+1}}=\frac{n+1}{n!\cdot n}$$

$n=6$ $\qquad\qquad$ $R_n<\frac{7}{6!\cdot 6}=\frac{7}{6!\cdot 6}<0{,}002$

$$y(-1)\approx 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}$$

$\frac{1}{2!}=0{,}5;$ $\quad\frac{1}{3!}=0{,}167$ (-) (с избытком); $\quad\frac{1}{4!}=0{,}042$ (-) (с избытком)

\+

$\frac{1}{5!}=0{,}008$ (+) (с недостатком)

$=2{,}717$

$2{,}717-0{,}001<y(-1)<2{,}717+0{,}003$

$2{,}716<y(-1)<2{,}720$

Ответ: $\qquad y(-1)\approx 2{,}72$

6. $\quad f(x)=\begin{cases}1 & -4<x<-1\\ 2 & -1<x<4\end{cases}$

$$a_0=\frac{1}{4}\cdot\left(\int_{-4}^{-1}dx+\int_{-1}^{4}2dx\right)=\frac{1}{4}\cdot(-1+4+8+2)=\frac{13}{4}$$

$$a_n=\frac{1}{4}\cdot\left(\int_{-4}^{-1}\cos\frac{\pi nx}{4}dx+\int_{-1}^{4}2\cos\frac{\pi nx}{4}dx\right)=\frac{1}{4}\cdot\left(\frac{4}{\pi n}\cdot\sin\frac{\pi nx}{4}\Big|_{-4}^{-1}+\frac{8}{\pi n}\cdot\sin\frac{\pi nx}{4}\Big|_{-1}^{4}\right)=$$

$$=\frac{1}{\pi n}\cdot\left(-\sin\frac{\pi n}{4}+2\sin\frac{\pi n}{4}\right)=\frac{1}{\pi n}\cdot\sin\frac{\pi n}{4}$$

$$b_n=\frac{1}{4}\cdot\left(-\frac{4}{\pi n}\cdot\cos\frac{\pi nx}{4}\Big|_{-4}^{-1}-\frac{8}{\pi n}\cdot\cos\frac{\pi nx}{4}\Big|_{-1}^{4}\right)=$$

$$=\frac{1}{4}\cdot\left(-\frac{4}{\pi n}\cdot\cos\frac{\pi n}{4}+\frac{4}{\pi n}\cdot(-1)^n-\frac{8}{\pi n}\cdot(-1)^n+\frac{8}{\pi n}\cdot\cos\frac{\pi n}{4}\right)=\frac{1}{\pi n}\cdot\left(\cos\frac{\pi n}{4}+(-1)^{n+1}\right)$$

Ответ: $f(x)=\frac{13}{8}+\sum_{n=1}^{\infty}\frac{1}{\pi n}\cdot\sin\frac{\pi n}{4}\cdot\cos\frac{\pi nx}{4}+\frac{1}{\pi n}\cdot\left(\cos\frac{\pi n}{4}+(-1)^{n+1}\right)\cdot\sin\frac{\pi nx}{4}$

7. $\quad f(x)=\begin{cases}2x & 0<x<\frac{\pi}{3}\\ \pi-x & \frac{\pi}{3}<x<\pi\end{cases}$ $\qquad$ по косинусам

$$a_0=\frac{2}{\pi}\cdot\left(\int_0^{\pi/3}2x\,dx+\int_{\pi/3}^{\pi}(\pi-x)dx\right)=\frac{2}{\pi}\cdot\left(x^2\Big|_0^{\pi/3}-\frac{(\pi-x)^2}{2}\Big|_{\pi/3}^{\pi}\right)=\frac{2}{\pi}\cdot\left(\frac{\pi^2}{9}+\frac{2\pi^2}{9}\right)=\frac{2}{3}\pi$$

$$a_n=\frac{2}{\pi}\cdot\left(\int_0^{\pi/3}2x\cdot\cos nx\,dx+\int_{\pi/3}^{\pi}(\pi-x)\cdot\cos nx\,dx\right)=$$

$$=\frac{2}{\pi}\cdot\left(2x\cdot\frac{\sin nx}{n}\Big|_0^{\pi/3}+\frac{2}{n^2}\cdot\cos nx\Big|_0^{\pi/3}+(\pi-x)\cdot\frac{\sin nx}{n}\Big|_{\pi/3}^{\pi}-\frac{\cos nx}{n^2}\Big|_{\pi/3}^{\pi}\right)=$$

$$=\frac{2}{\pi}\cdot\left[\frac{2\pi}{3n}\cdot\sin\frac{n\pi}{3}+\frac{2}{n^2}\cdot\cos\frac{\pi n}{3}-\frac{2}{n^2}-\frac{2\pi}{3n}\cdot\sin\frac{n\pi}{3}-\frac{(-1)^n}{n^2}+\frac{\cos\frac{\pi n}{3}}{n^2}\right]=$$

$$=\frac{2}{\pi n^2}\cdot\left[3\cos\frac{\pi n}{3}-2-(-1)^n\right]$$

<u>**Ответ:**</u> $f(x)=\frac{\pi}{3}+\sum_{n=1}^{\infty}\frac{2}{\pi n^2}\cdot\left[3\cos\frac{\pi n}{3}-2-(-1)^n\right]\cdot\cos nx$

Вариант 4.

1. $\sum_{n=2}^{\infty}\frac{2^n\cdot(x+3)^n}{n\cdot(3\ln n+1)^2}$

$$\lim_{n\to\infty}\frac{2^{n+1}\cdot|x+3|^{n+1}\cdot n\cdot(3\ln n+1)^2}{(n+1)\cdot[3\ln(n+1)+1]^2\cdot 2^n\cdot|x+3|^n}=2\cdot|x+3|$$

$|x+3|<\frac{1}{2}$

$x+3=-\frac{1}{2}$ $\quad\sum_{n=2}^{\infty}\frac{(-1)^n}{n\cdot(3\ln n+1)^2}$ абсолютно сходится

$\sum_{n=2}^{\infty}\frac{(-1)^n}{n\cdot(3\ln n+1)^2}$ -- сходится, так как $\frac{1}{n\ln^2 n}>\frac{1}{n(3\ln n+1)^2}$

$x+3=\frac{1}{2}$ $\quad\sum_{n=2}^{\infty}\frac{1}{n(3\ln n+1)^2}$ - сходится

<u>**Ответ:**</u> $-\frac{7}{2}\le x\le-\frac{5}{2}$

2. $\cos\frac{\pi x}{4}$

$$\cos\frac{\pi x}{4}=\cos\frac{\pi}{4}\cdot[(x-1)+1]=\cos\frac{\pi}{4}(x-1)\cos\frac{\pi}{4}-\sin\frac{\pi}{4}(x-1)\sin\frac{\pi}{4}=$$

$$=\frac{\sqrt{2}}{2}\cdot\left[\cos\frac{\pi}{4}(x-1)-\sin\frac{\pi}{4}(x-1)\right]$$

$$\cos\frac{\pi}{4}(x-1)=1-\frac{1}{2!}\cdot\frac{\pi^2}{4^2}(x-1)^2+\frac{1}{4!}\cdot\frac{\pi^4}{4^4}(x-1)^4-\dots+(-1)^n\cdot\frac{1}{(2n)!}\cdot\frac{\pi^{2n}}{4^{2n}}(x-1)^{2n}+\cdots$$

$-\infty<x<+\infty$

$$\sin\frac{\pi}{4}(x-1)=\frac{\pi}{4}(x-1)-\frac{\pi^{3}}{4^{3}\cdot 3!}(x-1)^{3}+\dots+(-1)^{n}\cdot\frac{\pi^{2n-1}\cdot(x-1)^{2n-1}}{4^{2n-1}\cdot(2n-1)!}+\dots$$

$-\infty < x < +\infty$

Ответ: $$\cos\frac{\pi x}{4}=\frac{\sqrt{2}}{2}\cdot\sum_{n=0}^{\infty}(-1)^{\frac{n^{2}+n}{2}}\cdot\left(\frac{\pi}{4}\right)^{n}\cdot\frac{(x-1)^{n}}{n!}$$

3. $$\int_{0}^{1}\frac{arctg\frac{x^{3}}{2}}{x^{2}}dx$$

$$arctgx=x-\frac{x^{3}}{3}+\frac{x^{5}}{5}-\frac{x^{7}}{7}+\dots+(-1)^{n-1}\cdot\frac{x^{2n-1}}{2n-1}+\dots \qquad -1<x<1$$

$$arctg\frac{x^{3}}{2}=\frac{x^{3}}{2}-\frac{x^{9}}{2^{3}\cdot 3}+\frac{x^{15}}{2^{5}\cdot 5}-\dots+(-1)^{n-1}\cdot\frac{x^{3(2n-1)}}{2^{2n-1}(2n-1)}+\dots$$

$-\sqrt[3]{2}<x<\sqrt[3]{2}$

$$\frac{arctg\frac{x^{3}}{2}}{x^{2}}=\frac{x}{2}-\frac{x^{7}}{2^{3}\cdot 3}+\frac{x^{13}}{2^{5}\cdot 5}-\dots+(-1)^{n-1}\cdot\frac{x^{6n-5}}{2^{2n-1}\cdot(2n-1)}+\dots$$

$$\int_{0}^{x}\frac{arctg\frac{x^{3}}{2}}{x^{2}}dx=\frac{x^{2}}{2\cdot 2}-\frac{x^{8}}{2^{3}\cdot 3\cdot 8}+\frac{x^{14}}{2^{5}\cdot 5\cdot 14}-\dots+(-1)^{n-1}\cdot\frac{x^{6n-4}}{(6n-4)\cdot 2^{2n-1}\cdot(2n-1)}+\dots$$

$$\int_{0}^{1}\frac{arctg\frac{x^{3}}{2}}{x^{2}}dx=\frac{1}{2\cdot 2}-\frac{1}{2^{3}\cdot 3\cdot 8}+\frac{1}{2^{5}\cdot 5\cdot 14}-\dots+(-1)^{n-1}\cdot\frac{1}{2^{2n-1}\cdot(2n-1)(6n-4)}+\dots$$

$$|R_{3}|<\frac{1}{2^{5}\cdot 5\cdot 14}<0{,}0005$$

$$\int_{0}^{1}\frac{arctg\frac{x^{3}}{2}}{x^{2}}dx=\frac{1}{4}-\frac{1}{2^{3}\cdot 3\cdot 8}$$

$\frac{1}{4}=0{,}25$

$-$

$\frac{1}{2^{3}\cdot 3\cdot 8}=0{,}005$ (+) с недостатком

$0{,}245$

Ответ: $$\int_{0}^{1}\frac{arctg\frac{x^{3}}{2}}{x^{2}}dx\approx 0{,}245$$

4. $y'=2x^{2}+y^{3}-5 \qquad y|_{x=2}=1$

$$y(x)=y(2)+\frac{y'(2)}{1!}\cdot(x-2)+\frac{y''(2)}{2!}\cdot(x-2)^{2}+\frac{y'''(2)}{3!}\cdot(x-2)^{3}+\dots$$

$y(2)=1$

$y'(2) = $ [illegible] $= 4$ $\qquad\qquad y' = 2x^2 + y^3 - 5$

$y''(2) = 8$ [illegible] $4 = 20$ $\qquad\qquad y'' = 4x + 3y^2 \cdot y'$

$y'''(2) = 4 + 6\cdot(4)^2 + 3\cdot 20 =$ $\qquad\qquad y''' = 4 + 6y\cdot(y')^2 + 3y^2 \cdot y''$

$= 4 + 6\cdot 16 + 60 = 160$

<u>Ответ:</u> $\qquad y(x) = 1 + 4(x-2) + \frac{20}{2!}(x-2)^2 + \frac{160}{3!}(x-2)^3 + \ldots$

5. $(x^2-3)y'' - 2xy' + 2y = (x^2-1)\cdot shx - 2xchx$

$$y(0) = 0 \qquad\qquad y'(0) = 2$$

$$-\quad shx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$+\quad x^2 shx = x^3 + \frac{x^5}{3!} + \ldots + \frac{x^{2n+1}}{(2n-1)!} + \ldots$$

$$-\quad 2xchx = 2x + \frac{2x^3}{2!} + \frac{2x^5}{4!} + \ldots + \frac{2x^{2n+1}}{(2n)!} + \ldots$$

$$-3x + x^3\cdot\left(1 - \frac{1}{3!} - \frac{2}{2!}\right) + x^5\cdot\left(\frac{1}{3!} - \frac{1}{5!} - \frac{2}{4!}\right) + \cdots + x^{2n+1}\cdot\left[\frac{1}{(2n-1)!} - \frac{1}{(2n+1)!} - \frac{2}{(2n)!} + \cdots\right]$$

$$\begin{array}{c|l}
2\text{-}2 & (x^2-1)\cdot shx - 2xchx = \sum\limits_{n=0}^{\infty} \frac{4n^2-2n-3}{(2n+1)!}\cdot x^{2n+1} \\
+2 & y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n + \cdots \\
-2 & xy'(x) = a_1x + 2a_2x^2 + 3a_3x^3 + \cdots + na_n\cdot x^n + \cdots \\
3 & y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots \\
+1 & x^2y''(x) = 2a_2x^2 + 3\cdot 2a_3x^3 + \cdots + n(n-1)a_nx^n + \cdots
\end{array}$$

$$(2a_0 - 6a_2) - 3\cdot 2\cdot 3a_3x - 3\cdot 4\cdot 3a_4x^2 + (2a_3 - 3\cdot 5\cdot 4a_5)x^3 + \ldots +$$

$$+ (2a_n - 2na_n - 3(n+2)(n+1)\cdot a_{n+2} + n(n-1)\cdot a_n)\cdot x^n + \ldots =$$

$$= \sum_{n=0}^{\infty}\left[(n-1)(n-2)a_n - 3(n+2)(n+1)a_{n+2}\right]\cdot x^n$$

$$\sum_{n=0}^{\infty}\left[(n-1)(n-2)a_n - 3(n+2)(n+1)a_{n+2}\right]\cdot x^n = \sum_{m=0}^{\infty}\frac{4m^2-2m-3}{(2m+1)!}\cdot x^{2m+1}$$

$$a_0 = 0 \qquad\qquad a_1 = 2$$

$$2a_0 - 6a_2 = 0 \qquad\qquad a_2 = 0$$

$$-3\cdot 3\cdot 2a_3 = -3 \qquad\qquad a_3 = \frac{1}{2\cdot 3} = \frac{1}{3!}$$

....

$$(2m+1)(2m-2)\cdot a_{2m} - 3(2m+2)(2m+1)\cdot a_{2m+2} = 0$$

$$a_{2m} = 0$$

$$(2m+1-1)(2m+1-2)\cdot a_{2m+1} - 3(2m+3)(2m+2)\cdot a_{2m+3} = \frac{4m^2-2m-3}{(2m+1)!}$$

Пусть $a_{2m+1}=\dfrac{1}{(2m+1)!}$

$$-3\cdot(2m+2)(2m+3)\cdot a_{2m+3}=\frac{4m^2-2m-3}{(2m+1)!}-\frac{2m(2m-1)}{(2m+1)!}$$

$$a_{2m+3}=\frac{-3}{(2m+1)!(-3)\cdot(2m+2)(2m+3)}=\frac{1}{(2m+3)!}$$

$$\boxed{y(x)=2x+\sum_{m=1}^{\infty}\frac{1}{(2m+1)!}\cdot x^{2m+1}}$$

$$y(1)=2+\sum_{m=1}^{\infty}\frac{1}{(2m+1)!};\qquad \frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\dots$$

$$R=\frac{1}{5!}+\frac{1}{7!}+\dots<\frac{1}{5!}\cdot\left(1+\frac{1}{6^2}+\frac{1}{6^4}+\dots\right)=\frac{1}{5!}\cdot\frac{1}{1-\frac{1}{36}}=\frac{36}{5!\cdot 35}<0{,}009$$

$$y(1)\approx 2+\frac{1}{3!}\approx 2{,}17$$

Ответ: $y(1)\approx 2{,}17$

6. $f(x)=\begin{cases}1 & -3<x<-2\\ 0 & -2<x<1\\ 2 & 1<x<3\end{cases}$

$$a_0=\frac{1}{3}\cdot\left(\int_{-3}^{-2}dx+\int_{1}^{3}2dx\right)=\frac{1}{3}\cdot(-2+3+6-2)=\frac{5}{3}$$

$$a_n=\frac{1}{3}\cdot\left(\int_{-3}^{-2}\cos\frac{\pi nx}{3}dx+\int_{1}^{3}2\cos\frac{\pi nx}{3}dx\right)=\frac{1}{3}\text{[illegible]}\sin\frac{\pi nx}{3}\Big|_{-3}^{-2}+\frac{6}{\pi n}\cdot\sin\frac{\pi nx}{3}\Big|_{1}^{3}\Big)=$$

$$=\frac{1}{\pi n}\cdot\left(-\sin\frac{2\pi n}{3}-2\sin\frac{\pi n}{3}\text{[illegible]}\frac{\pi n}{3}\right)$$

$$b_n=\frac{1}{3}\cdot\left(\int_{-3}^{-2}\sin\frac{\pi nx}{3}dx+\int_{1}^{3}2\text{[illegible]}\left(-\frac{3}{\pi n}\cos\frac{\pi nx}{3}\Big|_{-3}^{-2}-\frac{6}{\pi n}\cos\frac{\pi nx}{3}\Big|_{1}^{3}\right)=$$

$$=-\frac{1}{\pi n}\cdot\left(\cos\frac{2\pi n}{3}-(-1)^n+2\cdot(-1)^n-2\cos\frac{\pi n}{3}\right)=-\frac{1}{\pi n}\cdot\left[(-1)^n+\cos\frac{2\pi n}{3}-2\cos\frac{\pi n}{3}\right]$$

Ответ: $f(x)=\dfrac{5}{6}+\displaystyle\sum_{n=1}^{\infty}\frac{1}{\pi n}\left(-\sin\frac{2\pi n}{3}-2\sin\frac{\pi n}{3}\right)\cdot\cos\frac{\pi nx}{3}+$

$$+\frac{1}{\pi n}\cdot\left(2\cos\frac{\pi n}{3}-\cos\frac{2\pi n}{3}-(-1)^n\right)\cdot\sin\frac{\pi nx}{3}$$

7. $f(x)=\begin{cases}2x & 0<x<\frac{\pi}{3}\\ \pi-x & \frac{\pi}{3}<x<\pi\end{cases}$ по синусам

$$b_n = \frac{2}{\pi}\cdot\left(\int_0^{\pi/3} 2x\cdot\sin nx dx + \int_{\pi/3}^{\pi}(\pi - x)\cdot\sin nx dx\right) =$$

$$= \frac{2}{\pi}\cdot\left(-2x\cdot\frac{\cos nx}{n}\Bigg|_0^{\pi/3} + \frac{2\sin nx}{n^2}\Bigg|_0^{\pi/3} - (\pi - x)\cdot\frac{\cos nx}{n}\Bigg|_{\pi/3}^{\pi} - \frac{\sin nx}{n^2}\Bigg|_{\pi/3}^{\pi}\right) =$$

$$= \frac{2}{\pi}\cdot\left(-\frac{2\pi}{3n}\cos\frac{n\pi}{3} + \frac{2}{n^2}\sin\frac{\pi n}{3} + \frac{2\pi}{3n}\cos\frac{\pi n}{3} + \frac{1}{n^2}\sin\frac{\pi n}{3}\right) = \frac{6}{\pi n^2}\sin\frac{\pi n}{3}$$

Ответ: $f(x) = \sum_{n=1}^{\infty}\frac{6}{\pi n^2}\sin\frac{\pi n}{3}\sin nx$

Вариант 5.

1. $\sum_{n=1}^{\infty}\frac{(x-5)^n}{3^n\cdot\left(\sqrt[5]{n} - \sin\frac{1}{n}\right)}$

$$\lim_{n\to\infty}\frac{|x-5|^{n+1}\cdot 3^n\cdot\left(\sqrt[5]{n} - \sin\frac{1}{n}\right)}{3^{n+1}\cdot\left(\sqrt[5]{n+1} - \sin\frac{1}{n+1}\right)\cdot|x-5|^n} = \lim_{n\to\infty}\frac{|x-5|\left(\sqrt[5]{n} - \sin\frac{1}{n}\right)}{3\left(\sqrt[5]{n+1} - \sin\frac{1}{n+1}\right)} = \frac{|x-5|}{3}$$

$|x-5| < 3$

$x - 5 = 3$ $\qquad \sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n} - \sin\frac{1}{n}}$; $\qquad \frac{1}{\sqrt[5]{n} - \sin\frac{1}{n}} \sim \frac{1}{\sqrt[5]{n}}$ -- ряд расходится

$x - 5 = -3$ $\qquad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[5]{n} - \sin\frac{1}{n}}$ -- ряд сходится условно

$$\frac{\sqrt[5]{n} - \sin\frac{1}{n}}{\sqrt[5]{n+1} - \sin\frac{1}{n+1}} = \sqrt[5]{\frac{n}{n+1}}\cdot\frac{\left(1 - \frac{1}{\sqrt[5]{n}}\cdot\sin\frac{1}{n}\right)}{\left(1 - \frac{1}{\sqrt[5]{n+1}}\cdot\sin\frac{1}{n+1}\right)} < 1$$

Ответ: $2 \le x < 8$

2. xe^{2x} по степеням $(x+1)$

$xe^{2x} = [(x+1) - 1]\cdot e^{2(x+1)}\cdot e^{-2}$

$$e^{2(x+1)} = 1 + 2(x+1) + \frac{2^2(x+1)^2}{2!} + \ldots + \frac{2^n(x+1)^n}{n!} + \ldots$$

$$(x+1)\cdot e^{2(x+1)} = (x+1) + \frac{2(x+1)^2}{1!} + \cdots + \frac{2^{n-1}\cdot(x+1)^n}{(n-1)!} + \cdots$$

$-\infty < x < +\infty$

$$xe^{2x} = e^{-2}\cdot\left[-1 - (x+1) + \left(2 - \frac{2^2}{2!}\right)\cdot(x+1)^2 + \ldots + \left(\frac{2^{n-1}}{(n-1)!} - 2^n\cdot\frac{1}{n!}\right)\cdot(x+1)^n + \ldots\right]$$

<u>**Ответ:**</u> $xe^{2x} = e^{-2}\cdot\sum_{n=1}^{\infty}\frac{2^{n-1}(n-2)}{n!}(x+1)^n$

$-\infty < x < +\infty$

3. $\int_0^3 \frac{1}{x}\cdot\left(ch\frac{x}{3}-\cos\frac{x}{3}\right)dx$

$ch\frac{x}{3} = 1+\frac{x^2}{3^2\cdot 2!}+\frac{x^4}{3^4\cdot 4!}+\dots+\frac{x^{2n}}{3^{2n}\cdot(2n)!}+\dots$

$-\infty < x < +\infty$

$\cos\frac{x}{3} = 1-\frac{x^2}{3^2\cdot 2!}+\frac{x^4}{3^4\cdot 4!}-\dots+\frac{(-1)^n\cdot x^{2n}}{3^{2n}\cdot(2n)!}+\dots$

$-\infty < x < +\infty$

$ch\frac{x}{3}-\cos\frac{x}{3} = 2\cdot\frac{x^2}{3^2\cdot 2!}+2\cdot\frac{x^6}{3^6\cdot 6!}+\dots+\frac{2x^{2(2n-1)}}{3^{2(2n-1)}\cdot[2(2n-1)!]}+\dots$

$\frac{1}{x}\cdot\left(ch\frac{x}{3}-\cos\frac{x}{3}\right) = \frac{2x}{3^2\cdot 2!}+\frac{2x^5}{3^6\cdot 6!}+\dots+\frac{2x^{4n-3}}{3^{4n-2}\cdot(4n-2)!}+\dots$

$-\infty < x < +\infty$

$f(0) = 0!$

$\int_0^3 \frac{1}{x}\cdot\left(ch\frac{x}{3}-\cos\frac{x}{3}\right)dx = \frac{2\cdot 3^2}{3^2\cdot 2!\cdot 2}+\frac{2\cdot 3^6}{3^6\cdot 6!\cdot 6}+\dots+\frac{2\cdot 3^{4n-2}}{3^{4n-2}\cdot(4n-2)!\cdot(4n-2)}+\dots$

$\int_0^3 \frac{1}{x}\cdot\left(ch\frac{x}{3}-\cos\frac{x}{3}\right)dx = \sum_{n=1}^{\infty}\frac{1}{(2n-1)(4n-2)!}$

$|R| = \frac{1}{3\cdot 6!}+\frac{1}{5\cdot 10!}+\dots < 0{,}001$

<u>**Ответ:**</u> $\int_0^3 \frac{1}{x}\cdot\left(ch\frac{x}{3}-\cos\frac{x}{3}\right)dx \approx \frac{1}{2!} = 0{,}500$

4. $y' = e^{2y}+4x$ $\qquad y|_{x=-1} = 0$

$y(x) = y(-1)+\frac{y'(-1)}{1!}\cdot(x+1)+\frac{y''(-1)}{2!}\cdot(x+1)^2+\frac{y'''(-1)}{3!}\cdot(x+1)^3+\dots$

$y(-1) = 0$

$y' = e^{2y}+4x$ $\qquad y'(-1) = e^0-4 = -3$

$y'' = e^{2y}\cdot 2y'+4$ $\qquad y''(-1) = -6+4 = -2$

$y''' = e^{2y}\cdot(3y')^2+e^{2y}\cdot 2y''$ $\qquad y'''(-1) = [2\cdot(-3)]^2-4 = 32$

<u>**Ответ:**</u> $y(x) = -3\cdot(x+1)-\frac{2}{2!}\cdot(x+1)^2+\frac{32}{3!}\cdot(x+3)^3+\dots$

5. $(x^2+2)y''+4xy'+2y = 12x$ $\qquad y(0) = 2;$ $\qquad y'(0) = 0$

2 | $y(x) = a_0+a_1x+a_2x^2+\dots+a_nx^n+\dots$

4 | $xy'(x) = a_1x+2a_2x^2+\dots+na_n\cdot x^n+\dots$

$$
\begin{array}{r|l}
2 & y''(x) = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3a_4 x^2 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots \\
+1 & x^2 y''(x) = 2a_2 x^2 + \cdots + n(n-1)a_n x^n + \cdots \\
\hline
\end{array}
$$

$$(x^2+2)y'' + 4xy + 2y = (2a_0 + 4a_2) + (6a_1 + 3\cdot 2\cdot 3a_3)x +$$

$$+ (12a_2 + 4\cdot 3\cdot 2a_4)x^2 + \ldots + [(n+2)(n+1)\cdot a_n + 2(n+2)(n+1)\cdot a_{n+2}]\cdot x^n + \ldots$$

$a_0 = 2$ $\qquad\qquad$ $a_1 = 0$

$2a_0 + 4a_2 = 0$ $\qquad\qquad$ $a_2 = -1$

$6a_1 + 12a_3 = 12$ $\qquad\qquad$ $a_3 = 1$

....

$(n+2)(n+1)a_n + 2(n+2)(n+1)a_{n+2} = 0$ $\qquad\qquad$ $n > 0$

$a_n + 2a_{n+2} = 0$ $\qquad\qquad$ $a_{n+2} = -\dfrac{1}{2}a_n$

Пусть $a_{2m} = \dfrac{(-1)^m}{2^{m-1}}$

$2a_{2(m+1)} + a_{2m} = 0$ $\qquad\qquad$ $a_{2(m+1)} = \dfrac{(-1)^{m+1}}{2^m}$

Если $a_{2m+1} = \dfrac{(-1)^{m+1}}{2^{m-1}}$, тогда $a_{2m+3} = \dfrac{(-1)^{m+2}}{2^m}$

$$y(x) = 2 - x^2 + x^3 + \frac{1}{2}x^4 - \frac{1}{2}x^5 - \ldots + \frac{(-1)^m}{2^{m-1}}x^{2m} + \frac{(-1)^{m+1}}{2^{m-1}}x^{2m+1} + \ldots$$

$$y(1) = 2 - 1 + 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^2} + \ldots$$

Ряд сходится

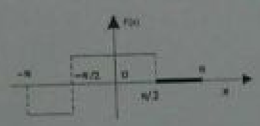
$$2 - 1 + 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^2} + \ldots + \frac{(-1)^m}{2^{m-1}} + \frac{(-1)^{m+1}}{2^{m-1}} = 2,$$

следовательно $y(1) = 2$

Ответ: $\quad y(x) = 2 - x^2 + x^3 + \dfrac{1}{2}x^4 - \dfrac{1}{2}x^5 - \ldots + \dfrac{(-1)^m}{2^{m-1}}x^{2m} + \dfrac{(-1)^{m+1}}{2^{m-1}}x^{2m+1} + \ldots$

$y(1) = 2$

6. $\quad f(x) = \begin{cases} -1 & -\pi < x < -\dfrac{\pi}{2} \\ 1 & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ 0 & \dfrac{\pi}{2} < x < \pi \end{cases}$



$$a_0 = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{-\pi/2} -1\cdot dx + \int_{-\pi/2}^{\pi/2} dx\right] = \frac{1}{\pi}\cdot\left[\frac{\pi}{2} - \pi + \pi\right] = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{-\pi/2} -\cos nx dx + \int_{-\pi/2}^{\pi/2} \cos nx dx\right] = \frac{1}{\pi n}\cdot\left[\sin\frac{\pi n}{2} + 2\sin\frac{n\pi}{2}\right] =$$

$$=\frac{3}{\pi n}\cdot\sin\frac{\pi n}{2}=\begin{cases}\dfrac{3}{\pi\cdot(2m-1)}\cdot(-1)^{m-1} & n=2m-1\\ 0 & n=2m\end{cases}$$

$$b_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{-\pi/2}-\sin nx dx+\int_{-\pi/2}^{\pi/2}\sin nx dx\right]=\frac{1}{\pi n}\cdot\left[\cos\frac{\pi n}{2}-(-1)^n\right]$$

Ответ: $f(x)=\frac{1}{4}+\sum_{n=1}^{\infty}\frac{3}{\pi n}\cdot\sin\frac{\pi n}{2}\cdot\cos nx+\frac{1}{\pi n}\left[\cos\frac{n\pi}{2}-(-1)^n\right]\cdot\sin nx$

7. $f(x)=\begin{cases}x & 0<x<2\\ 4-x & 2<x<5\end{cases}$ по косинусам

$$a_0=\frac{2}{5}\cdot\left[\int_0^2 x dx+\int_2^5(4-x)dx\right]=\frac{2}{5}\cdot\left[\frac{x^2}{2}\bigg|_0^2-\frac{(4-x)^2}{2}\bigg|_2^5\right]=\frac{2}{5}\cdot\left(2-\frac{1}{2}+2\right)=\frac{2}{5}\cdot\frac{7}{2}=\frac{7}{5}$$

$$a_n=\frac{2}{5}\cdot\left[\int_0^2 x\cos\frac{\pi nx}{5}dx+\int_2^5(4-x)\cos\frac{\pi nx}{5}dx\right]=\frac{2}{5}\cdot\left[x\frac{5}{\pi n}\sin\frac{\pi nx}{5}\bigg|_0^2+\frac{5^2}{\pi^2n^2}\cos\frac{\pi nx}{5}\bigg|_0^2+\right.$$

$$\left.+(4-x)\cdot\frac{5}{\pi n}\cdot\sin\frac{\pi nx}{5}\bigg|_2^5-\frac{5^2}{\pi^2n^2}\cdot\cos\frac{\pi nx}{5}\bigg|_2^5\right]=\frac{2}{5}\cdot\left[\frac{10}{\pi n}\cdot\sin\frac{2\pi n}{5}+\frac{5^2}{\pi^2n^2}\cdot\cos\frac{2\pi n}{5}-1-\right.$$

$$\left.-\frac{10}{\pi n}\cdot\sin\frac{2\pi n}{5}-\frac{5^2}{\pi^2n^2}\cdot(-1)^n+\frac{5^2}{\pi^2n^2}\cdot\cos\frac{2\pi n}{5}\right]=\frac{10}{\pi^2n^2}\cdot\left[2\cos\frac{2\pi n}{5}+(-1)^{n+1}-1\right]$$

Ответ: $f(x)=0{,}7+\sum_{n=1}^{\infty}\frac{10}{\pi^2n^2}\cdot\left[2\cos\frac{2\pi n}{5}-1+(-1)^{n+1}\right]\cdot\cos\frac{\pi nx}{5}$

Вариант 6.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\dfrac{6^n\cdot\left(x-\frac{5}{6}\right)^n}{\sqrt[5]{n}-\sin\frac{1}{n}}$

$$\lim_{n\to\infty}\frac{6^{n+1}\cdot\left|x-\frac{5}{6}\right|^{n+1}\cdot\left(\sqrt[5]{n}-\sin\frac{1}{n}\right)}{\left(\sqrt[5]{n+1}-\sin\frac{1}{n+1}\right)\cdot6^n\cdot\left|x-\frac{5}{6}\right|^n}=\lim_{n\to\infty}\frac{6\cdot\left|x-\frac{5}{6}\right|^{n+1}\cdot\sqrt[5]{n}\cdot\left(1-\sin\frac{1}{n}\right)}{\sqrt[5]{n+1}\cdot\left(1-\frac{1}{\sqrt[5]{n+1}}\cdot\sin\frac{1}{n+1}\right)}=6\cdot\left|x-\frac{5}{6}\right|$$

$6\left|x-\frac{5}{6}\right|<1 \qquad \left|x-\frac{5}{6}\right|<\frac{1}{6}$

$x-\frac{5}{6}=-\frac{1}{6} \quad \sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n}-\sin\frac{1}{n}}$ расходится, так как $\frac{1}{\sqrt[5]{n}-\sin\frac{1}{n}}\sim\frac{1}{\sqrt[5]{n}}$

$x-\frac{5}{6}=\frac{1}{6} \quad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[5]{n}-\sin\frac{1}{n}}$ - сходится условно

$$\frac{\sqrt[5]{n}-\sin\frac{1}{n}}{\sqrt[5]{n+1}-\sin\frac{1}{n+1}}=\sqrt[5]{\frac{n}{n+1}}\cdot\left(\frac{1-\frac{1}{\sqrt[5]{n}}\cdot\sin\frac{1}{n}}{1-\frac{1}{\sqrt[5]{n+1}}\cdot\sin\frac{1}{n+1}}\right)<1$$

Ответ: $\frac{2}{3}<x\le$

2. $\ln 4x \qquad a=3$

$$\ln[4\cdot(x-3)+12]=\ln\left[12\cdot\left(1+\frac{x-3}{3}\right)\right]=\ln 12+\ln\left[1+\frac{x-3}{3}\right]=\ln 12+\frac{x-3}{3}-\frac{(x-3)^2}{3^2\cdot 2}+$$
$$+\frac{(x-3)^3}{3^3\cdot 3}-\ldots+(-1)^{n-1}\cdot\frac{(x-3)^n}{3^n\cdot n}+\ldots$$

$$\left|\frac{x-3}{3}\right|<1 \qquad 0<x\le 6$$

Ответ: $\ln 4x=\ln 12+\frac{x-3}{3}-\frac{(x-3)^2}{3^2\cdot 2}+\frac{(x-3)^3}{3^3\cdot 3}-\ldots+(-1)^{n-1}\cdot\frac{(x-3)^n}{3^n\cdot n}+\ldots$

$0<x\le 6$

3. $\int_0^2\frac{1}{x^2}\left(ch\frac{x}{2}-\cos\frac{x}{2}\right)dx$

$$ch\frac{x}{2}=1+\frac{x^2}{2^2\cdot 2!}+\frac{x^4}{2^4\cdot 4!}+\cdots+\frac{x^{2n}}{2^{2n}\cdot(2n)!}+\cdots \qquad -\infty<x<+\infty$$

$$\cos\frac{x}{2}=1-\frac{x^2}{2^2\cdot 2!}+\frac{x^4}{2^4\cdot 4!}-\cdots+(-1)^n\cdot\frac{x^{2n}}{2^{2n}\cdot(2n)!}+\cdots \qquad -\infty<x<+\infty$$

$$ch\frac{x}{2}-\cos\frac{x}{2}=\sum_{n=1}^{\infty}\frac{x^{2(2n-1)}}{2^{4n-3}\cdot[2\cdot(2n-1)]!}$$

$$\frac{1}{x^2}\left(ch\frac{x}{2}-\cos\frac{x}{2}\right)=\sum_{n=1}^{\infty}\frac{x^{4n-4}}{2^{4n-3}\cdot[2\cdot(2n-1)]!}$$

$$\int_0^2\frac{1}{x^2}\cdot\left(ch\frac{x}{2}-\cos\frac{x}{2}\right)dx=\sum_{n=1}^{\infty}\frac{2^{4n-3}}{2^{4n-3}\cdot(4n-3)\cdot[4n-2]!}$$

$$|R|<\frac{1}{5\cdot 6!}+\frac{1}{9\cdot 10!}+\frac{1}{13\cdot 14!}+\ldots<\frac{1}{5\cdot 6!}\cdot\left(1+\frac{1}{9\cdot 7^4}+\frac{1}{9\cdot 7^8}+\ldots\right)=$$
$$=\frac{1}{5\cdot 6!}\cdot\frac{1}{1-\frac{1}{9\cdot 7^4}}=\frac{1}{5\cdot 6!}\cdot\frac{31609}{31608}<0{,}0003$$

Ответ: $\int_0^2\frac{1}{x^2}\left(ch\frac{x}{2}-\cos\frac{x}{2}\right)dx\approx\frac{1}{2!}=0{,}500$

4. $y'=x^3-e^{-x} \qquad y|_{x=2}=0$

$$y(x)=y(2)+\frac{y'(2)}{1!}\cdot(x-2)+\frac{y''(2)}{2!}\cdot(x-2)^2+\frac{y'''(2)}{3!}\cdot(x-2)^3+\cdots$$

$$y' = x^3 - e^{-y} \qquad y(2)=0$$
$$y'(2) = 7$$
$$y'' = 3x^2 + e^{-y}\cdot y' \qquad y''(2) = 12 + 7 = 19$$
$$y''' = 6x - e^{-y}\cdot(y')^2 + e^{-y}y'' \qquad y'''(2) = 12 - 49 + 19 = -18$$

Ответ: $y(x) = 7\cdot(x-2) + \frac{19}{2!}\cdot(x-2)^2 - \frac{18}{3!}\cdot(x+3)^3 + \cdots$

5. $(4x^2+1)y'' + 16xy' + 8y = 6x \qquad y(0) = 2; \qquad y'(0) = 0$

$$\begin{array}{r|l} 8 & y(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n + \cdots \\ 16 & xy' = a_1x + 2a_2x^2 + \ldots + na_n\cdot x^n + \cdots \\ 1 & y'' = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots \\ 4 & x^2y'' = 2a_2x^2 + \cdots + n(n-1)a_nx^n + \cdots \end{array}$$

$$(4x^2+1)y'' + 16xy' + 8y = (8a_0 + 2a_2) + (24a_1 + 3\cdot 2a_3)x +$$
$$+ (48a_2 + 4\cdot 3a_4)x^2 + \ldots + [4\cdot(n+1)(n+2)\cdot a_n + (n+2)(n+1)\cdot a_{n+2}]\cdot x^n + \ldots$$

$a_0 = 2 \qquad a_1 = 0$

$8a_0 + 2a_2 = 0 \qquad a_2 = -8$

$24a_1 + 6a_3 = 6 \qquad a_3 = 1$

$48a_2 + 12a_4 = 0 \qquad a_4 = 32$

....

$4\cdot(n+1)(n+2)a_n + (n+2)(n+1)a_{n+2} = 0 \qquad a_{n+2} = -4a_n$

$n > 1.$ Пусть $a_{2n} = 2\cdot(-4)^n$,

тогда $a_{2n+2} = 2\cdot(-4)^{n+1}$

Пусть $a_{2n+1} = (-4)^{n-1}$, тогда $a_{2n+3} = (-4)^n$.

$y(x) = 2 - 8x^2 + x^3 + 32x^4 - \ldots + (-1)^n\cdot 2\cdot 4^n x^{2n} + (-1)^{n-1}\cdot 4^{n-1}\cdot x^{2n+1} + \ldots$

$x = \frac{1}{4}$

$$y\left(\frac{1}{4}\right) = 2 - 8\cdot\frac{1}{4^2} + \frac{1}{4^3} + 32\cdot\frac{1}{4^4} - \ldots + (-1)^n\cdot\frac{2\cdot 4^n}{4^{2n}} + (-1)^{n-1}\cdot\frac{4^{n-1}}{4^{2n+1}} + \ldots$$

ряд абсолютно сходится.

$$y\left(\frac{1}{4}\right) = 2 - \frac{2\cdot 4}{4^2} + \frac{2\cdot 4^2}{4^4} - \ldots + (-1)^n\cdot\frac{2\cdot 4^n}{4^{2n}} + \ldots + \frac{1}{4^3} - \frac{1}{4^4} + \frac{1}{4^5} - \ldots =$$

$$\frac{2}{1+\frac{1}{4}} + \frac{\frac{1}{4^3}}{1+\frac{1}{4}} = \frac{8}{5} + \frac{1}{5\cdot 4^2} = \frac{129}{80} \approx 1{,}61$$

Ответ: $y(x) = 2 - 8x^2 + x^3 + 32x^4 - \ldots + (-1)^n\cdot 2\cdot 4^n\cdot x^{2n} + (-1)^{n-1}\cdot 4^{n-1}\cdot x^{2n+1} + \ldots$

$y\left(\frac{1}{4}\right) = 1{,}61$

6. $$f(x)=\begin{cases}0 & -\pi<x<-\dfrac{\pi}{2}\\ 1 & -\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\\ -1 & \dfrac{\pi}{2}<x<\pi\end{cases}$$

$$a_0=\frac{1}{\pi}\cdot\left[\int_{-\pi/2}^{\pi/2}dx+\int_{\pi/2}^{\pi}dx\right]=\frac{1}{\pi}\cdot\left(\pi-\frac{\pi}{2}\right)=\frac{1}{2}$$

$$a_n=\frac{1}{\pi}\cdot\left[\int_{-\pi/2}^{\pi/2}\cos nxdx-\int_{\pi/2}^{\pi}\cos nxdx\right]=\frac{1}{\pi}\cdot\left[\frac{2}{n}\cdot\sin\frac{\pi n}{2}+\frac{1}{n}\cdot\sin\frac{\pi n}{2}\right]=\frac{3}{\pi n}\cdot\sin\frac{\pi n}{2}$$

$$b_n=\frac{1}{\pi}\cdot\left[\int_{-\pi/2}^{\pi/2}\sin nxdx+\int_{-\pi/2}^{\pi}\sin nxdx\right]=\frac{1}{\pi}\cdot\left[\frac{1}{n}\cdot\cos nx\Big|_{-\pi/2}^{\pi}\right]=\frac{1}{\pi n}\cdot\left[(-1)^n-\cos\frac{\pi n}{2}\right]$$

Ответ: $$f(x)=\frac{1}{4}+\sum_{n=1}^{\infty}\frac{3}{n\pi}\cdot\sin\frac{\pi n}{2}\cos nx=\frac{1}{\pi n}\cdot\left[(-1)^n-\cos\frac{\pi n}{2}\right]$$

7. $$f(x)=\begin{cases}x & 0<x<2\\ 4-x & 2<x<5\end{cases}\quad \text{по синусам}$$

$$b_n=\frac{2}{5}\cdot\left[\int_0^2 x\sin\frac{\pi nx}{5}dx+\int_2^5(4-x)\sin\frac{\pi nx}{5}dx\right]=\frac{2}{5}\cdot\left[-x\cdot\frac{5}{\pi n}\cdot\cos\frac{\pi nx}{5}\Big|_0^2+\frac{5^2}{\pi^2n^2}\times\right.$$

$$\left.\times\sin\frac{\pi nx}{5}\Big|_0^2-(4-x)\cdot\frac{5}{\pi n}\cdot\cos\frac{\pi nx}{5}\Big|_2^5-\frac{5^2}{\pi^2n^2}\cdot\sin\frac{\pi nx}{5}\Big|_2^5\right]=\frac{2}{5}\cdot\left[-\frac{10}{\pi n}\cdot\cos\frac{2\pi n}{5}+\right.$$

$$+\frac{5^2}{\pi^2n^2}\cdot\sin\frac{2\pi n}{5}+\frac{5}{\pi n}\cdot(-1)^n+$$

$$\left.+\frac{10}{\pi n}\cdot\cos\frac{2\pi n}{5}+\frac{5^2}{\pi^2n^2}\cdot\sin\frac{2\pi n}{5}\right]=\frac{20}{\pi^2n^2}\cdot\sin\frac{2\pi n}{5}+\frac{2}{\pi n}\cdot(-1)^n$$

Ответ: $$f(x)=\sum_{n=1}^{\infty}\left[\frac{20}{\pi^2n^2}\cdot\sin\frac{2\pi n}{5}+\frac{2}{\pi n}\cdot(-1)^n\right]\cdot\sin\frac{\pi nx}{5}$$

Вариант 7.

1. $$\sum_{n=1}^{\infty}\frac{4^n\cdot\left(x+\frac{3}{4}\right)^n}{\sqrt[4]{n+4}-4^{-n}}$$

$$\lim_{n\to\infty}\frac{4^{n+1}\cdot\left|x+\frac{3}{4}\right|^{n+1}\cdot\left(\sqrt[4]{n+4}-4^{-n}\right)}{\left[\sqrt[4]{n+5}-4^{-(n+1)}\right]\cdot4^n\cdot\left|x+\frac{3}{4}\right|^n}=\lim_{n\to\infty}\frac{4\cdot\left|x+\frac{3}{4}\right|\cdot\sqrt[4]{n+4}\cdot\left(1-\frac{1}{4^n\cdot\sqrt[4]{n+4}}\right)}{\sqrt[4]{n+5}\cdot\left(1-\frac{1}{4^{n+1}\cdot\sqrt[4]{n+5}}\right)}=4\cdot\left|x+\frac{3}{4}\right|$$

$$4\cdot\left|x+\frac{3}{4}\right|<\frac{1}{4}$$

$x+\frac{3}{4}=\frac{1}{4}$ $\quad \sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n+4}-4^{-n}}$ расходится, т.к. $\frac{1}{\sqrt[4]{n+4}-4^{-n}}\sim\frac{1}{\sqrt[4]{n+4}}$

$x+\frac{3}{4}=-\frac{1}{4}$ $\quad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n+4}-4^{-n}}$ сходится условно

$$\frac{\sqrt[4]{n+4}\cdot\left(1-\frac{1}{4^n\cdot\sqrt[4]{n+4}}\right)}{\sqrt[4]{n+5}\cdot\left(1-\frac{1}{4^{n+1}\cdot\sqrt[4]{n+5}}\right)}=\sqrt[4]{\frac{n+4}{n+5}}\cdot\left(\frac{1-\frac{1}{4^n\cdot\sqrt[4]{n+4}}}{1-\frac{1}{4^{n+1}\cdot\sqrt[4]{n+5}}}\right)<1$$

Ответ: $-1\le x<-\frac{1}{2}$

2. $\sin\frac{x}{2}$ $\qquad a=-\frac{\pi}{2}$

$$\sin\left[\frac{x+\frac{\pi}{2}}{2}-\frac{\pi}{4}\right]=\sin\frac{\left(x+\frac{\pi}{2}\right)}{2}\cdot\frac{\sqrt{2}}{2}-\cos\frac{\left(x+\frac{\pi}{2}\right)}{2}\cdot\frac{\sqrt{2}}{2}$$

$$\sin\frac{x+\frac{\pi}{2}}{2}=\frac{x+\pi/2}{2}-\frac{\left(x+\frac{\pi}{2}\right)^3}{2^3\cdot3!}+\dots+(-1)^{n-1}\cdot\frac{\left(x+\frac{\pi}{2}\right)^{2n-1}}{2^{2n-1}\cdot(2n-1)!}+\dots$$

$-\infty<x<+\infty$

$$\cos\frac{x+\pi/2}{2}=1-\frac{\left(x+\frac{\pi}{2}\right)^2}{2^2\cdot2!}+\dots+(-1)^n\cdot\frac{\left(x+\frac{\pi}{2}\right)^{2n}}{2^{2n}\cdot(2n)!}+\dots$$

$-\infty<x<+\infty$

Ответ: $\sin\frac{x}{2}=-\frac{\sqrt{2}}{2}\cdot\sum_{n=0}^{\infty}(-1)^{\frac{n^2+n}{2}}\cdot\frac{\left(x+\frac{\pi}{2}\right)^n}{2^n\cdot n!}$

$-\infty<x<+\infty$

3. $\int_0^{2/3}e^{-x^4}dx=?$

$$e^{-x^4}=1-x^4+\frac{x^8}{2!}-\frac{x^{12}}{3!}-\dots+(-1)^n\cdot\frac{x^{4n}}{n!}+\dots \qquad -\infty<x<+\infty$$

$$\int_0^x e^{-x^4}dx=x-\frac{x^5}{5}+\frac{x^9}{2!\,9}-\frac{x^{13}}{3!\,13}+\dots+(-1)^n\cdot\frac{x^{4n+1}}{n!\,(4n+1)}+\dots \qquad -\infty<x<+\infty$$

$$\int_0^{2/3}e^{-x^4}dx=\frac{2}{3}-\frac{\left(\frac{2}{3}\right)^5}{5}+\frac{\left(\frac{2}{3}\right)^9}{2!\,9}-\frac{\left(\frac{2}{3}\right)^{13}}{3!\,13}+\dots$$

$$|R|<\frac{\left(\frac{2}{3}\right)^{13}}{3!\,13}<0{,}0001$$

+	$\frac{2}{3}=$ [illegible]	(-)	с избытком
-	$\frac{1}{5}\cdot\left(\frac{2}{3}\right)^5=0{,}0263$	(+)	с недостатком
+	$\frac{1}{18}\cdot\left(\frac{2}{3}\right)^9=0{,}0014$	(+)	с недостатком
	0,6418		

$$0{,}6417<\int_0^{2/3} e^{-x^4}\,dx<0{,}6420$$

<u>**Ответ:**</u> $\int_0^{2/3} e^{-x^4}\,dx\approx 0{,}642$

4. $y'=x^2+\frac{2}{y}-6$ $\qquad y|_{x=3}=-1$

$$y(x)=y(3)+\frac{y'(3)}{1!}\cdot(x-3)+\frac{y''(3)}{2!}\cdot(x-3)^2+\frac{y'''(3)}{3!}\cdot(x-3)^3+\ldots$$

$y(3)=-1$

$y'=x^2+\frac{2}{y}-6$ $\qquad y'(3)=9-2-6=1$

$y''=2x-\frac{2y'}{y^2}$ $\qquad y''(3)=6-2=4$

$y'''=2+\frac{4}{y^3}\cdot(y')^2-\frac{2}{y^2}\cdot y''$ $\qquad y'''(3)=2-4-8=-10$

<u>**Ответ:**</u> $y(x)=-1+(x-3)+\frac{4}{2!}\cdot(x-3)^2-\frac{10}{3!}\cdot(x-3)^3+\cdots$

5. $(x^2-1)y''-2xy'+2y=2\ln(1+x)+\frac{1-3x}{1+x}$ $\qquad y(0)=0;\qquad y'(0)=1$

2	$y(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+\cdots$
-2	$xy'=a_1x+2a_2x^2+3a_3x^3+\ldots+na_n\cdot x^n+\cdots$
-1	$y''=2a_2+3\cdot2a_3x+4\cdot3a_4x^2+5\cdot4a_5x^3+\cdots+(n+2)(n+1)a_{n+2}x^n+\cdots$
+1	$x^2y''=2a_2x^2+3\cdot2a_3x^3+\cdots+n(n-1)a_nx^n+\cdots$

$$(x^2-1)y''-2xy'+2y=(2a_0-2a_2)-3\cdot2a_3x-4\cdot3a_4x^2+(2a_3-5\cdot4a_5)x^3+\ldots+$$
$$+[(n-1)(n-2)\cdot a_n-(n+2)(n+1)\cdot a_{n+2}]\cdot x^n+\ldots$$

$2\ln(1+x)=2x-\frac{2x^2}{2}+\frac{2x^3}{3}-\ldots+(-1)^{n-1}\cdot\frac{2x^n}{n}+\ldots$ $\qquad -1<x\le1$

$\frac{1}{1+x}=1-x+x^2-x^3+\ldots+(-1)^nx^n+\ldots$

$-\frac{3x}{1+x}=-3x+3x^2-3x^3+\ldots+(-1)^n\cdot3x^n+\ldots$

$\frac{1-3x}{1+x}=1-4x+4x^2-4x^3+\ldots+(-1)^n\cdot4x^n+\ldots$ $\qquad -1<x<1$

$$2\ln(1+x)+\frac{1-3x}{1+x}=1-2x+3x^2+\left(\frac{2}{3}-4\right)x^3+\ldots+(-1)^{n-1}\cdot\left(\frac{2}{n}-4\right)x^n+\ldots$$

$a_0=0$ $\qquad a_1=1$

$2a_0-2a_2=1$ $\qquad a_2=-\frac{1}{2}$

$-3\cdot 2a_3=-2$ $\qquad a_3=\frac{1}{3}$

$-4\cdot 3a_4=3$ $\qquad a_3=-\frac{1}{4}$

....

$$(n-1)(n-2)a_n-(n+2)(n+1)a_{n+2}=(-1)^{n-1}\cdot\frac{2-4n}{n}$$

Пусть $a_n=\frac{(-1)^{n-1}}{n}$,

тогда - $(n+2)(n+1)a_{n+2}=(-1)^{n-1}\cdot\left[\frac{2-4n}{n}-\frac{(n-1)(n-2)}{n}\right]$

$$a_{n+2}=(-1)^{n+1}\cdot\frac{n^2+n}{n(n+2)(n+1)}=(-1)^{n+1}\cdot\frac{1}{n+2}$$

$$y(x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots+(-1)^{n-1}\cdot\frac{x^n}{n}+\ldots \qquad -1<x<1;\quad y(x)=\ln x$$

$$y\left(-\frac{1}{5}\right)=-\frac{1}{5}-\frac{1}{5^2\cdot 2}-\frac{1}{5^3\cdot 3}-\frac{1}{5^4\cdot 4}-\ldots$$

$$|R|=\frac{1}{5^3\cdot 3}+\frac{1}{5^4\cdot 4}+\ldots<\frac{1}{5^3\cdot 3}\cdot\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots\right)<0{,}004$$

$$y\left(-\frac{1}{5}\right)\approx-0{,}20-0{,}02=-0{,}22$$

Ответ: $\quad y(x)=\ln[x+1]$

$\qquad y\left(-\frac{1}{5}\right)=-0{,}22$

6. $\quad f(x)=\begin{cases}\pi & -\pi<x<0\\ \pi-2x & 0<x<\pi\end{cases}$

$$a_0=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{0}\pi\,dx+\int_{0}^{\pi}(\pi-2x)dx\right]=\frac{1}{\pi}\cdot\left[\pi^2-\frac{(\pi-2x)^2}{4}\Bigg|_0^\pi\right]=$$

$$=\frac{1}{\pi}\cdot\left(\pi^2-\frac{\pi^2}{4}+\frac{\pi^2}{4}\right)=\pi$$

$$a_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{0}\pi\cos nx dx+\int_{0}^{\pi}(\pi-2x)\cos nx dx\right]=\frac{1}{\pi}\cdot\left[\frac{\pi}{n}\sin nx\Bigg|_{-\pi}^{0}+\right.$$

$$\left.+(\pi-2x)\cdot\frac{\sin nx}{n}\Bigg|_0^\pi-2\frac{\cos nx}{n^2}\Bigg|_0^\pi\right]=-\frac{2}{\pi n^2}\cdot\left[(-1)^n-1\right]=\frac{2}{\pi n^2}\cdot\left[1-(-1)^n\right]$$

$$= \frac{1}{\pi} \cdot \left[\int_{-\pi}^{0} \pi \sin nx dx + \int_{0}^{\pi} (\pi - 2x) \sin nx dx \right] = \frac{1}{\pi} \cdot \left[-\frac{\pi}{n} \cos nx \Big|_{-\pi}^{0} - \right.$$

[illegible] $\left. \cos nx \Big|_0^\pi - \frac{2}{n^2} \sin nx \Big|_0^\pi \right] = \frac{1}{\pi} \cdot \left[-\frac{\pi}{n} + \frac{\pi}{n} \cdot (-1)^n + \frac{\pi}{n} \cdot (-1)^n + \frac{\pi}{n} \right] = (-1)^n \cdot \frac{2}{n}$

[illegible] $\frac{\pi}{\ldots} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2} \left[1 - (-1)^n \right] \cos nx + (-1)^n \cdot \frac{2}{n} \sin nx$

7. $f(x) = \begin{cases} 0 & 0 < x < 1 \\ 1 & 1 < x < 2 \\ 2 & 2 < x < 3 \end{cases}$ по косинусам

$$a_0 = \frac{2}{3} \cdot \left(\int_1^2 dx + \int_2^3 2dx \right) = \frac{2}{3} \cdot (1 + 2) = 2$$

$$a_n = \frac{2}{3} \cdot \left(\int_1^2 \cos \frac{\pi n x}{3} dx + \int_2^3 2 \cos \frac{\pi n x}{3} dx \right) = \frac{2}{3} \cdot \left(\frac{3}{\pi n} \cdot \sin \frac{n \pi x}{3} \Big|_1^2 + \frac{2 \cdot 3}{\pi n} \sin \frac{n \pi x}{3} \Big|_2^3 \right) =$$

$$= \frac{2}{3} \cdot \left(\frac{2}{\pi n} \sin \frac{2n\pi}{3} - \frac{3}{\pi n} \sin \frac{\pi n}{3} - \frac{6}{\pi n} \sin \frac{2\pi n}{3} \right) = \frac{2}{3} \cdot \left(-\frac{3}{\pi n} \sin \frac{2n\pi}{3} - \frac{3}{\pi n} \sin \frac{\pi n}{3} \right) =$$

$$= \frac{2}{3} \cdot \left[-(-1)^{n-1} \cdot \frac{3}{\pi n} \cdot \sin \frac{\pi n}{3} - \frac{3}{\pi n} \cdot \sin \frac{\pi n}{3} \right] = \frac{4}{\pi (2m-1)} \cdot \sin \frac{\pi (2m-1)}{3},$$

так как $\sin \frac{2n\pi}{3} = \sin \left(\pi n - \frac{\pi n}{3} \right) = (-1)^{n-1} \cdot \sin \frac{\pi n}{3}$

<u>Ответ:</u> $f(x) = 1 + \sum_{m=1}^{\infty} \frac{4}{\pi (2m-1)} \cdot \sin \frac{\pi (2m-1)}{3} \cdot \cos \frac{n \pi x}{3}$

Вариант 8.

1. $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{9^n \cdot (x-1)^{2n}}{\sqrt[9]{n} - 9^{-n}}$

$$\lim_{n \to \infty} \frac{9^{n+1} \cdot |x-1|^{2n+2} \cdot \left(\sqrt[9]{n} - 9^{-n} \right)}{\left(\sqrt[9]{n+1} - 9^{-(n+1)} \right) \cdot 9^n \cdot |x-1|^{2n}} = \lim_{n \to \infty} \frac{9 \cdot |x-1|^2 \cdot \sqrt[9]{n} \cdot \left(1 - \frac{1}{9^n \cdot \sqrt[9]{n}} \right)}{\sqrt[9]{n+1} \cdot \left(1 - \frac{1}{9^{n+1} \cdot \sqrt[9]{n+1}} \right)} = 9 \cdot |x-1|^2$$

$|x - 1| < \frac{1}{3}$

$x - 1 = \pm \frac{1}{3}$ $\qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[9]{n} - 9^{-n}}$ сходится условно

$\lim_{n \to \infty} a_n = 0$

$$\frac{a_{n+1}}{a_n} = \sqrt[9]{\frac{n}{n+1}} \cdot \left(\frac{1 - \frac{1}{9^n \cdot \sqrt[9]{n}}}{1 - \frac{1}{9^{n+1} \cdot \sqrt[9]{n+1}}} \right)$$

Ответ: $\frac{2}{3} \le x \le \frac{4}{3}$

2. $(x-3)\cdot e^{\frac{x}{2}}$ в окрестности точки (-3)

$$(x-3)\cdot e^{\frac{x}{2}} = [(x+3)-6]\cdot e^{\frac{x+3}{2}}\cdot e^{-3/2}$$

$$e^{\frac{x+3}{2}} = 1 + \frac{(x+3)}{2} + \frac{(x+3)^2}{2^2\cdot 2!} + \ldots + \frac{(x+3)^n}{2^n\cdot n!} + \ldots \qquad -\infty < x < +\infty$$

$$(x+3)\cdot e^{\frac{x+3}{2}} = (x+3) + \frac{(x+3)^2}{2} + \ldots + \frac{(x+3)^n}{2^{n-1}\cdot (n-1)!} + \ldots$$

$$(x-3)\cdot e^{\frac{x}{2}} = -6e^{-3/2} + e^{-3/2}\cdot \sum_{n=1}^{\infty} \frac{(n-3)}{2^{n-1}\cdot n!}\cdot (x+3)^n \qquad (-\infty < x < +\infty)$$

Ответ: $f(x) = (x-3)\cdot e^{\frac{x}{2}} = -6e^{-\frac{3}{2}} + e^{-\frac{3}{2}}\cdot \sum_{n=1}^{\infty} \frac{(n-3)}{2^{n-1}\cdot n!}\cdot (x+3)^n$

$(-\infty < x < +\infty)$

3. $\int_0^1 \frac{dx}{\sqrt[3]{8+x^2}}$ Вычислить с точностью до 0,001

$$\frac{1}{\sqrt[3]{8+x^2}} = \frac{1}{2\cdot\sqrt[3]{1+\frac{x^2}{8}}} = \frac{1}{2}\cdot\left(1+\frac{x^2}{8}\right)^{-\frac{1}{3}} = \frac{1}{2}\cdot\left(1 - \frac{x^2}{3\cdot 8} + \frac{1\cdot 4}{3^2\cdot 8^2\cdot 2!}x^4 - \frac{1\cdot 4\cdot 7}{3^3\cdot 8^3\cdot 3!}x^6 + \ldots +\right.$$

$$\left. + (-1)^n\cdot\frac{1\cdot 4\cdot 7\cdot\ldots\cdot(3n-2)}{3^n\cdot 8^n\cdot n!\cdot(2n+1)} + \ldots\right) \qquad |R| < \frac{1\cdot 4}{2\cdot 3^2\cdot 8^2\cdot 2!\cdot 5} < 0{,}0005$$

$+$ $\frac{1}{2} = 0{,}5000$

$-$ $\frac{1}{2\cdot 3\cdot 8\cdot 3} = 0{,}0069$

0,4931

Ответ: $\int_0^1 \frac{dx}{\sqrt[3]{8+x^2}} \approx 0{,}493$

4. $y' = xy - \frac{1}{y} + x \qquad y(-1) = 1 \qquad y(x) = y(-1) + \frac{y'(-1)}{1!}\cdot(x+1) + \frac{y''(-1)}{2!}\cdot(x+1) + \ldots$

$y(-1) = 1$

$y' = xy - \frac{1}{y} + x \qquad y'(-1) = -1-1-1 = -3$

$y'' = y + xy' + \frac{y'}{y^2} + 1 \qquad y''(-1) = 1+3-3+1 = 2$

$y''' = 2y' + xy'' + \frac{y''y^2 - 2y(y')^2}{y^4} \qquad y'''(-1) = -6-2+2-18 = -24$

Ответ: $y(x) = 1 - 3\cdot(x+1) + \frac{2}{2!}\cdot(x+1)^2 - \frac{24}{3!}\cdot(x+1)^3 + \cdots$

5. $(x^2-1)y''-2xy'+2y = 2\ln(1-x)+\dfrac{3x+1}{1-x}$ $\qquad y(0)=0;\qquad y'(0)=-1$

$$\begin{array}{r|l} 2 & y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n + \dots \\ -2 & xy(x) = a_1x + 2a_2x^2 + 3a_3x^3 + \dots + na_n \cdot x^n + \dots \\ -1 & y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \dots + (n+2)(n+1)a_{n+2}x^n + \dots \\ 1 & x^2y'' = 2a_2x^2 + 3\cdot 2a_3x^3 + \dots + n(n-1)a_nx^n + \dots \end{array}$$

$$(x^2-1)y''-2xy'+2y = (2a_0-2a_2) - 3\cdot 2a_3x - 4\cdot 3a_4x^2 + \dots +$$
$$+[(n-1)(n-2)\cdot a_n - (n+2)(n+1)\cdot a_{n+2}]\cdot x^n + \dots$$

$$2\ln(1-x) = -2x - \frac{2x^2}{2} - \frac{2x^3}{3} - \dots - \frac{2x^n}{n} - \dots \qquad (-1 \le x < 1)$$

$$\frac{3x+1}{1-x} = \frac{3x}{1-x} + \frac{1}{1-x} = 1 + 4x + 4x^2 + \dots + 4x^n + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + \dots$$

$$\frac{3x}{1-x} = 3x + 3x^2 + \dots + 3x^n + \dots \qquad (-1 < x < 1)$$

$$2\ln(1-x) + \frac{3x+1}{1-x} = 1 - 2x + 3x^2 + \left(4-\frac{2}{3}\right)x^3 + \dots + \left(4-\frac{2}{n}\right)x^n + \dots$$

$\underline{a_0 = 0}$ $\qquad \underline{a_1 = -1}$

$2a_0 - 2a_2 = 1 \qquad\qquad a_2 = -\dfrac{1}{2}$

$-3\cdot 2a_3 = 2 \qquad\qquad a_3 = -\dfrac{1}{3}$

$-4\cdot 3a_4 = 3 \qquad\qquad a_4 = -\dfrac{1}{4}$

....

$(n-1)(n-2)a_n - (n+2)(n+1)a_{n+2} = \left(4-\dfrac{2}{n}\right) \qquad$ Пусть $a_n = -\dfrac{1}{n}$

$$(n+2)(n+1)a_{n+2} = -\frac{(n-1)(n-2)}{n} - \frac{(4n-2)}{n}$$

$$a_{n+2} = -\frac{n^2+n}{n(n+2)(n+1)} = -\frac{1}{n+2}$$

$$y(x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots - \frac{x^n}{n} + \dots = \ln(1-x) \qquad (-1 < x < 1)$$

$$y\left(\frac{1}{4}\right) = -\frac{1}{4} - \frac{1}{4^2\cdot 2} - \frac{1}{4^3\cdot 3} - \dots - \frac{1}{4^n\cdot n} - \dots$$

$$|R| = \frac{1}{4^3\cdot 3} + \frac{1}{4^4\cdot 5} + \dots < \frac{1}{4^3\cdot 3} + \frac{1}{4^4\cdot 3} + \dots = \frac{1}{4^3\cdot 3\cdot\left(1-\frac{1}{4}\right)} = \frac{1}{4^2\cdot 3^2} < 0{,}007$$

$$\frac{1}{4} = 0,250$$

$$\frac{1}{4^2 \cdot 2} = 0,031$$

$$0,281$$

$$y\left(\frac{1}{4}\right) \approx -0,28$$

Ответ: $y(x) = \ln(1-x)$; $y\left(\frac{1}{4}\right) \approx -0,28$

6. $f(x) = \begin{cases} \pi + 2x & -\pi < x < 0 \\ \pi & 0 < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\cdot\left(\int_{-\pi}^{0}(\pi+2x)dx + \int_{0}^{\pi}\pi dx\right) = \frac{1}{\pi}\cdot\left[\frac{(\pi+2x)^2}{4}\Big|_{-\pi}^{0} + \pi^2\right] = \frac{1}{\pi}\cdot\left[\frac{\pi^2}{4} - \frac{\pi^2}{4} + \pi^2\right] = \pi$$

$$a_n = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{0}(\pi+2x)\cos nx dx + \int_{0}^{\pi}\cos nx dx\right] = \frac{1}{\pi}\cdot\left[(\pi+2x)\cdot\sin\frac{nx}{n}\Big|_{-\pi}^{0} + \right.$$

$$\left. + \frac{2}{n^2}\cdot\cos nx\Big|_{-\pi}^{0}\right] = \frac{2}{\pi n^2}\cdot[1-(-1)^n]$$

$$b_n = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{0}(\pi+2x)\sin nx dx + \int_{0}^{\pi}\sin nx dx\right] = \frac{1}{\pi}\cdot\int_{-\pi}^{0}2x\sin nx dx =$$

$$= \frac{2}{\pi}\cdot\left[-x\cdot\frac{\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\sin nx}{n^2}\Big|_{-\pi}^{0}\right] = -\frac{2}{\pi n}\pi\cos n\pi = -\frac{2}{n}\cdot(-1)^n.$$

Ответ: $f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{\pi n^2}[1-(-1)^n]\cdot\cos nx + \frac{2}{n}\cdot(-1)^{n+1}\cdot\sin nx$

7. $f(x) = \begin{cases} 0 & 0 < x < 1 \\ 1 & 1 < x < 2 \\ 2 & 2 < x < 3 \end{cases}$ по синусам

$$b_n = \frac{2}{3}\cdot\left[\int_{1}^{2}\sin\frac{\pi nx}{3}dx + \int_{2}^{3}2\sin\frac{\pi nx}{3}dx\right] = = \frac{2}{3}\cdot\left[-\frac{3}{\pi n}\cdot\cos\frac{\pi nx}{3}\Big|_{1}^{2} - \frac{6}{\pi n}\cdot\cos\frac{\pi nx}{3}\Big|_{2}^{3}\right] =$$

$$= 2\cdot\left[-\frac{1}{\pi n}\cos\frac{2\pi n}{3} + \frac{1}{\pi n}\cdot\cos\frac{\pi n}{3} - \frac{2}{\pi n}\cdot(-1)^n + \frac{2}{\pi n}\cdot\cos\frac{2\pi n}{3}\right] =$$

$$= \frac{2}{\pi n}\cdot\left[\cos\frac{2\pi n}{3} + \cos\frac{\pi n}{3} - 2\cdot(-1)^n\right] = \frac{2}{\pi n}\cdot\left[(1-(-1)^n)\cdot\cos\frac{2\pi n}{3} - 2\cdot(-1)^n\right]$$

Ответ: $f(x) = \sum_{n=1}^{\infty}\frac{2}{\pi n}\cdot\left[(1-(-1)^n)\cdot\cos\frac{\pi n}{3} - 2\cdot(-1)^n\right]\cdot\sin\frac{\pi nx}{3}$

Вариант 19.

1. $\sum_{n=1}^{\infty}\frac{(x+1)^{2n-1}}{n+\ln(n+1)}$

$\lim_{n\to\infty}\frac{|x+1|^{2n+1}\cdot[n+\ln(n+1)]}{[(n+1)+\ln(n+2)]\cdot|x+1|^{2n-1}}=(x+1)^2;$ $|x+1|<1$

$x+1=1$ $\sum_{n=1}^{\infty}\frac{1}{n+\ln(n+1)}$ расходится (сравнить с $\sum_{n=1}^{\infty}\frac{1}{n}$)

$x+1=-1$ $\sum_{n=1}^{\infty}\frac{-1}{n+\ln(n+1)}$ расходится

__Ответ:__ $-2<x<0$

2. $f(x)=ch2x$ по степеням $x-1$

$ch2x=\frac{1}{2}(e^{2x}+e^{-2x})=\frac{e^2}{2}\cdot e^{2(x-1)}+\frac{1}{2e^2}\cdot e^{-2(x-1)}=$

$=\frac{e^2}{2}\sum_{n=0}^{\infty}\frac{2^n(x-1)^n}{n!}+\frac{1}{2e^2}\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^n(x-1)^n}{n!}=\frac{1}{2}\sum_{n=0}^{\infty}\left[e^2+\frac{(-1)^n}{e^2}\right]\cdot\frac{2^n(x-1)^n}{n!}$

$-\infty<x<+\infty$

__Ответ:__ $f(x)=\frac{1}{2}\sum_{n=0}^{\infty}\left[e^2+\frac{(-1)^n}{e^2}\right]\cdot\frac{2^n(x-1)^n}{n!}$

$-\infty<x<+\infty$

3. $\int_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx$ $\ln(1+x)=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{n+1}}{n+1}$

$\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)=\sum_{n=0}^{\infty}\frac{x^{3n+1}}{10^{n+1}\cdot(n+1)}\cdot(-1)^{2n+1}$

$\int_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx=-\sum_{n=0}^{\infty}\frac{x^{3n+2}}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\Bigg|_0^1=-\sum_{n=0}^{\infty}\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\approx$

$\approx-\left(\frac{1}{2\cdot10\cdot1}+\frac{1}{5\cdot100\cdot2}\right)\approx-0{,}051$

$|\delta|=\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}+\frac{1}{(3n+5)\cdot10^{n+2}\cdot(n+2)}+\ldots<\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\times$

$\times\left[1+\frac{1}{10}+\frac{1}{10^2}+\ldots\right]=\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)\cdot9}$

$n=2$ $\delta<\frac{1}{21600}$

__Ответ:__ $\int_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx\approx-0{,}051$

4. $y'=\sqrt{y}-x$ $x=1$ $y=4$

$y'=1$

$$y'' = \frac{y'}{2\sqrt{y}} - 1 = -\frac{3}{4}$$

$$y''' = \frac{1}{4} \cdot \frac{2yy'' - y'^2}{y\sqrt{y}} = -\frac{7}{32}$$

$$y^{IV} = \frac{1}{4} \cdot \frac{2y^2 y''' - 3yy' y'' + \frac{3}{2} y'^3}{y^2 \sqrt{y}} = \frac{7}{256}$$

Ответ: $y(x) = 4 + \frac{1}{1!} \cdot (x-1) - \frac{3}{4 \cdot 2!} \cdot (x-1)^2 - \frac{7}{32 \cdot 3!} \cdot (x-1)^3 + \frac{7}{256 \cdot 4!}(x-1)^4 + \ldots$

5. $(2x^2 - 1)y'' - 4xy' + 4y = (5 - 2x^2)\cos x + 4x \sin x$

$x = 0 \qquad y = 1; \qquad y' = 0 \qquad y|_{x=2} = ?$

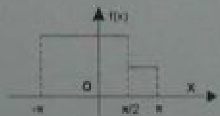
$$y = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + \ldots$$

$$y|_{x=2} = 1 - \frac{2^2}{2!} + \frac{2^4}{4!} - \frac{2^6}{6!} + \ldots + (-1)^n \cdot \frac{2^{2n}}{(2n)!} + \ldots \approx 1 - 2 + \frac{16}{24} - \frac{64}{720} \approx -0{,}42$$

$$|\delta| < \frac{256}{40320} < 0{,}01$$

Ответ: $y = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + \ldots$

$y|_{x=2} = -0{,}42$



6. $f(x) = \begin{cases} \pi & -\pi < x < \frac{\pi}{2} \\ \frac{\pi}{2} & \frac{\pi}{2} < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi} \cdot \left[\int_{-\pi}^{\pi/2} \pi \cdot dx + \int_{\pi/2}^{\pi} \frac{\pi}{2} dx \right] = \frac{7}{4}\pi$$

$$a_n = \frac{1}{\pi} \cdot \left[\int_{-\pi}^{\pi/2} \pi \cos nx dx + \int_{\pi/2}^{\pi} \frac{\pi}{2} \cos nx dx \right] = \frac{1}{\pi} \cdot \left[\frac{\pi}{n} \cdot \sin nx \Big|_{-\pi}^{\pi/2} + \frac{\pi}{2n} \cdot \sin nx \Big|_{\pi/2}^{\pi} \right] =$$

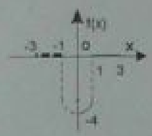
$$= \frac{1}{2n} \cdot \sin \frac{\pi n}{2}$$

$$b_n = \frac{1}{\pi} \cdot \left[\int_{-\pi}^{\pi/2} \pi \sin nx dx + \int_{\pi/2}^{\pi} \frac{\pi}{2} \sin nx dx \right] = \frac{1}{\pi} \cdot \left[-\frac{\pi}{n} \cos nx \Big|_{-\pi}^{\pi/2} - \frac{\pi}{2n} \cos nx \Big|_{\pi/2}^{\pi} \right] =$$

$$= \frac{1}{2n} \cdot \left[(-1)^n - \cos \frac{n\pi}{2} \right]$$

Ответ: $f(x) = \frac{7}{8}\pi + \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{1}{n} \cdot \sin \frac{n\pi}{2} \cdot \cos nx + \frac{1}{n} \cdot \left[(-1)^n - \cos \frac{n\pi}{2} \right] \cdot \sin nx$

7. $f(x) = \begin{cases} x^2 - 4 & 0 < x < 1 \\ 0 & 1 < x < 3 \end{cases}$ по косинусам



$$a_0 = \frac{2}{3}\int_0^1 (x^2-4)dx = -\frac{22}{9}$$

$$a_0 = \frac{2}{3}\cdot\int_0^1 (x^2-4)\cos\frac{n\pi x}{3}dx =$$

$$= \frac{2}{3}\cdot\left[\frac{3x^2}{\pi n}\sin\frac{n\pi x}{3} + \frac{18x}{\pi^2 n^2}\cos\frac{n\pi x}{3} - \frac{54}{\pi^3 n^3}\sin\frac{n\pi x}{3} - \frac{12}{\pi n}\sin\frac{n\pi x}{3}\right]_0^1 =$$

$$= \frac{2}{n\pi}\cdot\left[\frac{6}{\pi n}\cos\frac{n\pi}{3} - \frac{18}{n^2\pi^2}\sin\frac{n\pi}{3} - 3\sin\frac{n\pi}{3}\right]$$

Ответ: $f(x) = -\frac{11}{9} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{6}{\pi n}\cos\frac{n\pi}{3} - \frac{18}{n^2\pi^2}\sin\frac{n\pi}{3} - 3\sin\frac{n\pi}{3}\right]\cdot\cos\frac{n\pi x}{3}$

Вариант 20.

1. $\sum_{n=1}^{\infty}\frac{(x-1)^{2n-1}}{\sqrt{n}+\ln(n+1)}$

$\lim_{n\to\infty}\frac{|x-1|^{2n+1}\cdot[\sqrt{n}+\ln(n+1)]}{[\sqrt{n+1}+\ln(n+2)]|x-1|^{2n-1}} = |x-1|^2$ $\qquad |x-1|<1$

$x-1=1 \qquad \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\ln(n+1)}$ расходится (сравнить с $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$)

$x-1=-1 \qquad \sum_{n=1}^{\infty}\frac{-1}{\sqrt{n}+\ln(n+1)}$ расходится

Ответ: $0<x<2$

2. $f(x) = sh3x$ по степеням $x+1$

$ch3x = \frac{1}{2}(e^{3x}-e^{-3x}) = \frac{1}{2e^3}\cdot e^{3(x+1)} - \frac{e^3}{2}\cdot e^{-3(x+1)} =$

$= \frac{1}{2e^3}\sum_{n=0}^{\infty}\frac{3^n(x+1)^n}{n!} - \frac{e^3}{2}\sum_{n=0}^{\infty}(-1)^n\cdot\frac{3^n(x+1)^n}{n!} = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{e^3}-(-1)^n e^3\right]\cdot\frac{3^n(x+1)^n}{n!}$

$-\infty<x<+\infty$

Ответ: $f(x) = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{e^3}-(-1)^n\cdot e^3\right]\cdot\frac{3^n\cdot(x+1)^n}{n!}$

$-\infty<x<+\infty$

3. $\int_0^1 \ln\left(1-\frac{x^5}{5}\right)dx \qquad \ln(1+x) = \sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{n+1}}{n+1}$

$\ln\left(1-\frac{x^5}{5}\right) = \sum_{n=0}^{\infty}(-1)^{2n+1}\cdot\frac{x^{5n+5}}{5^{n+1}\cdot(n+1)};$

$\int_0^1 \ln\left(1-\frac{x^5}{5}\right)dx = -\sum_{n=0}^{\infty}\frac{x^{5n+6}}{(5n+6)\cdot 5^{n+1}\cdot(n+1)}\Bigg|_0^1 = -\sum_{n=0}^{\infty}\frac{1}{(5n+6)\cdot 5^{n+1}\cdot(n+1)} \approx$

Вариант 9.

1. $\sum_{n=1}^{\infty} \frac{2^n \cdot \left(x-\frac{1}{2}\right)^n}{2n^3+2^{-n}}$

$$\lim_{n\to\infty} \frac{2^{n+1} \cdot \left|x-\frac{1}{2}\right|^{n+1} \cdot (2n^3+2^{-n})}{[2\cdot(n+1)^3+2^{-(n+1)}]\cdot 2^n \cdot \left|x-\frac{1}{2}\right|^n} = \lim_{n\to\infty} \frac{2\cdot\left|x-\frac{1}{2}\right|\cdot n^3\cdot\left(2+\frac{1}{n^3\cdot 2^n}\right)}{(n+1)^3\cdot\left[2+\frac{1}{(n+1)^3\cdot 2^{n+1}}\right]} = 2\cdot\left|x-\frac{1}{2}\right|$$

$\left|x-\frac{1}{2}\right| < \frac{1}{2}$

$x-\frac{1}{2}=\frac{1}{2}$ $\sum_{n=1}^{\infty}\frac{1}{2n^3+2^{-n}}$ схо

дится, т.к. $\frac{1}{2n^3+2^{-n}} \sim \frac{1}{2n^3}$

$x-\frac{1}{2}=-\frac{1}{2}$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n^3+2^{-n}}$ сходится условно

<u>Ответ:</u> $0 \le x \le 1$

2. $\ln(4+x)$ в окрестности точки 1

$$\ln(4+x)=\ln[5+(x-1)]=\ln 5\cdot\left[1+\frac{x-1}{5}\right]=\ln 5+\frac{x-1}{5}-\frac{(x-1)^2}{5^2\cdot 2}+$$

$$+\frac{(x-1)^3}{5^3\cdot 3}-\ldots+(-1)^{n+1}\cdot\frac{(x-1)^n}{5^n\cdot n}+\ldots$$

$-4 < x \le 6$

<u>Ответ:</u> $\ln(4+x)=\ln 5+\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{(x-1)^n}{5^n\cdot n}$

$-4 < x \le 6$

3. $\int_0^2 e^{\frac{x^3}{100}}dx$ с точностью, меньшей 0,001

$$e^{\frac{x^3}{100}}=1+\frac{x^3}{100}+\frac{x^6}{100^2\cdot 2!}+\cdots+\frac{x^{3n}}{100^n\cdot n!}+\cdots \qquad -\infty < x < +\infty$$

$$\int_0^2 e^{\frac{x^3}{100}}dx = 2+\frac{2^4}{100\cdot 4}+\frac{2^7}{100^2\cdot 2!\cdot 7}+\ldots+\frac{2^{3n+1}}{100^n\cdot n!\cdot(3n+1)}+\cdots$$

$$|R|=\frac{2^7}{100^2\cdot 14}+\frac{2^{10}}{100^3\cdot 3!\cdot 10}+\ldots<\frac{2^7}{100^2\cdot 14}\cdot\left[1+\left(\frac{2^3}{100}\right)\cdot\frac{1}{3}+\left(\frac{2^3}{100}\right)^2\cdot\frac{1}{3^2}+\ldots\right]=$$

$$=\frac{2^7}{100^2\cdot 14}\cdot\frac{1}{1-\frac{8}{100\cdot 3}}=\frac{2^7\cdot 100\cdot 3}{100^2\cdot 14\cdot(100\cdot 3-8)}<0,001$$

Ответ: $\int_0^2 e^{\frac{x^3}{100}}dx \approx 2+0{,}040 = 2{,}040$

4. $y' = x - y + \dfrac{x}{y}$ $\qquad y|_{x=2} = 1$

$$y(x) = y(2) + \frac{y'(2)}{1!}\cdot(x-2) + \frac{y''(2)}{2!}\cdot(x-2)^2 + \frac{y'''(2)}{3!}\cdot(x-2)^3 + \ldots$$

$y' = x - y + \dfrac{x}{y}$ $\qquad y'(2) = 2 - 1 + 2 = 3$

$y'' = 1 - y' + \dfrac{y - xy'}{y^2}$ $\qquad y''(2) = 1 - 3 + \dfrac{1-6}{1} = -7$

$y''' = -y'' + \dfrac{-y''xy^2 - 2yy'(y - xy')}{y^4}$ $\qquad y'''(2) = 7 + (14 + 6\cdot 5) = 51$

Ответ: $y(x) = 1 + \dfrac{3}{1!}\cdot(x-2) - \dfrac{7}{2!}\cdot(x-2)^2 + \dfrac{51}{3!}\cdot(x-2)^3 + \ldots$

5. $(x^2-3)y'' - 2xy' + 2y = (5 - x^2)\cos x + 2x\sin x$

$y(0) = 1; \qquad y'(0) = 0$

2	$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n + \cdots$
-2	$xy' = a_1x + 2a_2x^2 + 3a_3x^3 + \ldots + na_n\cdot x^n + \cdots$
-3	$y'' = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots$
1	$x^2y'' = 2a_2x^2 + 3\cdot 2a_3x^3 + \cdots + n(n-1)a_nx^n + \cdots$

$$(x^2-3)y'' - 2xy' + 2y = (2a_0 - 6a_2) - 3\cdot 2\cdot 3a_3x - 3\cdot 4\cdot 3a_4x^2 +$$
$$+ (2a_3 + 5\cdot 4a_5)x^3 + \ldots + [(n-1)(n-2)\cdot a_n + (-3)\cdot(n+2)(n+1)\cdot a_{n+2}]\cdot x^n + \ldots$$

5	$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + (-1)^n\cdot\dfrac{x^{2n}}{(2n)!} + \ldots$
-1	$x^2\cos x = x^2 - \dfrac{x^4}{2!} + \ldots + (-1)^{n-1}\cdot\dfrac{x^{2n}}{(2n-2)!} + \ldots$
2	$x\sin x = x^2 - \dfrac{x^4}{3!} + \cdots + (-1)^{n-1}\cdot\dfrac{x^{2n}}{(2n-1)!} + \cdots$

$$(5 - x^2)\cos x + 2x\sin x = 5 - \frac{3}{2}x^2 + \frac{9}{4!}x^4 - \ldots + \frac{(-1)^n}{(2n)!}\cdot(4n^2 - 6n + 5)x^{2n} + \ldots$$

$a_0 = 1$ $\qquad a_1 = 0$

$2a_0 - 6a_2 = 5$ $\qquad a_2 = -\dfrac{1}{2}$

$-3\cdot 2\cdot 3a_3 = 0$ $\qquad a_3 = 0$

$-3\cdot 4\cdot 3a_4 = -\dfrac{3}{2}$ $\qquad a_4 = \dfrac{1}{4!}$

....

$a_{2n-1} = 0$

Пусть $a_{2n} = (-1)^n \cdot \frac{1}{(2n)!}$

$$(2n-1)(2n-2)a_{2n} - 3\cdot(2n+2)(2n+1)\cdot a_{2n+2} = (-1)^n \cdot \frac{4n^2-6n+5}{(2n)!}$$

$$-3\cdot(2n+2)(2n+1)\cdot a_{2n+2} = (-1)^n \cdot \frac{4n^2-6n+5-4n^2+6n-2}{(2n)!}$$

$$a_{2n+2} = (-1)^n \cdot \frac{3}{(2n)!\cdot(-3)\cdot(2n+1)(2n+2)}$$

$$a_{2n+2} = (-1)^{n+1} \cdot \frac{1}{(2n+2)!}$$

$$y(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \ldots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + \ldots$$

$y(x) = \cos x$

$y(2)$ вычислить с точностью 0,01

$$y(2) = 1 - \frac{1}{2!}\cdot 2^2 + \frac{1}{4!}\cdot 2^4 - \ldots + (-1)^n \cdot \frac{2^{2n}}{(2n)!} + \ldots$$

$$|R| < \frac{2^8}{8!} < 0{,}007$$

$$y(2) \approx 1 - 2 + \frac{1}{4!}\cdot 2^4 - \frac{1}{6!}\cdot 2^6 \approx -1 + \frac{2}{3} - \frac{2^3}{3\cdot 5\cdot 6} = -1 + \frac{2}{3} - \frac{2^3}{9\cdot 10} =$$

$$= -1 + \frac{60-8}{9\cdot 10} \approx -1 + 0{,}578 = -0{,}422$$

Ответ: $y(x) = \cos x$

$y(2) = -0{,}422$

6. $$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 3 & -\frac{\pi}{2} < x < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\cdot \int_{-\pi/2}^{\pi/4} 3dx = \frac{3}{\pi}\cdot\left(\frac{\pi}{4}+\frac{\pi}{2}\right) = \frac{9}{4}$$

$$a_n = \frac{1}{\pi}\cdot \int_{-\pi/2}^{\pi/4} 3\cos nx dx = \frac{3}{\pi}\cdot \frac{\sin nx}{n}\Bigg|_{-\pi/2}^{\pi/4} = \frac{3}{\pi n}\cdot\left[\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{1}{\pi}\cdot \int_{-\pi/2}^{\pi/4} 3\sin nx dx = -\frac{3}{\pi n}\cos nx\Bigg|_{-\pi/2}^{\pi/4} = -\frac{3}{\pi n}\cdot\left[\cos\frac{n\pi}{4} - \cos\frac{n\pi}{2}\right]$$

Ответ:

$$f(x) = \frac{9}{8} + \sum_{n=1}^{\infty} \frac{3}{\pi n}\cdot\left[\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right]\cdot\cos nx + \frac{3}{\pi n}\cdot\left[\cos\frac{n\pi}{2} - \cos\frac{n\pi}{4}\right]\cdot\sin nx$$

7. $f(x)=\begin{cases}1+x & 0<x<1\\ 3-x & 1<x<3\end{cases}$ по синусам

$$b_n=\frac{2}{3}\cdot\left[\int_0^1(1+x)\cdot\sin\frac{n\pi x}{3}dx+\int_1^3(3-x)\cdot\sin\frac{n\pi x}{3}dx\right]=$$

$$=\frac{2}{3}\cdot\left[-(1+x)\cdot\frac{3}{\pi n}\cdot\cos\frac{n\pi x}{3}\Big|_0^1+\frac{3}{\pi n}\int_0^1\cos\frac{\pi nx}{3}dx-\frac{3}{\pi n}\cdot(3-x)\cdot\cos\frac{n\pi x}{3}\Big|_1^3-\right.$$

$$\left.-\frac{3}{\pi n}\int_1^3\cos\frac{n\pi x}{3}dx\right]=\frac{2}{3}\cdot\left[-\frac{6}{n\pi}\cdot\cos\frac{n\pi}{3}+\frac{3}{\pi n}+\frac{9}{\pi^2 n^2}\cdot\sin\frac{n\pi x}{3}\Big|_0^1+\frac{6}{n\pi}\cdot\cos\frac{n\pi}{3}-\right.$$

$$\left.-\frac{9}{\pi^2 n^2}\cdot\sin\frac{n\pi x}{3}\Big|_1^3\right]=\frac{2}{3}\cdot\left[\frac{3}{\pi n}+\frac{18}{\pi^2 n^2}\cdot\sin\frac{n\pi}{3}\right]$$

__Ответ:__ $f(x)=\sum_{n=1}^{\infty}\frac{2}{\pi n}\cdot\left[1+\frac{6}{\pi n}\cdot\sin\frac{\pi n}{3}\right]\cdot\sin\frac{\pi nx}{3}$

Вариант 10.

1. $\sum_{n=1}^{\infty}\frac{3^n\cdot\left(x-\frac{1}{3}\right)^n}{2^{2n}\cdot\left(\sqrt{n}-2^{-n}\right)}$

$$\lim_{n\to\infty}\frac{3^{n+1}\cdot\left|x-\frac{1}{3}\right|^{n+1}\cdot 2^{2n}\cdot(\sqrt{n}-2^{-n})}{2^{2n+2}\cdot(\sqrt{n+1}-2^{-(n+1)})\cdot 3^n\cdot\left|x-\frac{1}{3}\right|^n}=\frac{3}{2^2}\cdot\left|x-\frac{1}{3}\right|$$

$\left|x-\frac{1}{3}\right|<\frac{4}{3}$ $\boxed{-1\le x<\frac{5}{3}}$

$x-\frac{1}{3}=\frac{4}{3}$ $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}-2^{-n}}$ - расходится

$x-\frac{1}{3}=-\frac{4}{3}$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}-2^{-n}}$ - сходится условно

т.к. $\begin{matrix}a_n\to 0\\ n\to\infty\end{matrix}$ и

$$\frac{|a_{n+1}|}{|a_n|}=\frac{\sqrt{n}-\frac{1}{2^n}}{\sqrt{n+1}-2^{\frac{1}{n+1}}}=\sqrt{\frac{n}{n+1}}\cdot\left(\frac{1-\frac{1}{2^n\cdot\sqrt{n}}}{1-\frac{1}{2^{n+1}\cdot\sqrt{n+1}}}\right)<1$$

2. $\frac{3}{x^2-x-2}$ по степеням $(x+2)$

$$\frac{3}{(x+1)(x-2)}=-\frac{1}{x+1}+\frac{1}{x-2}$$

$$-\frac{1}{x+1}=\frac{-1}{(x+2)-1}=\frac{1}{1-(x+2)}=\sum_{n=0}^{\infty}(x+2)^n \qquad |x+2|<1$$

$$\frac{1}{x-2}=\frac{1}{(x+2)-4}=-\frac{1}{4}\cdot\frac{1}{1-\frac{x+2}{4}}=-\frac{1}{4}\cdot\sum_{n=0}^{\infty}\frac{(x+2)^n}{4^n} \qquad |x+2|<4$$

$$\frac{3}{(x+1)(x-2)}=\sum_{n=0}^{\infty}\left(1-\frac{1}{4^{n+1}}\right)\cdot(x+2)^n \qquad |x+2|<1$$

$$\boxed{-3<x<-1}$$

3. $\int_0^2 e^{\frac{x^3}{12}}dx$

$$e^{\frac{x^3}{12}}=1+\frac{x^3}{12}+\frac{x^6}{12^2\cdot 2!}+\frac{x^9}{12^3\cdot 3!}+\dots+\frac{x^{3n}}{12^n\cdot n!}+\dots \qquad -\infty<x<+\infty$$

$$\int_0^2 e^{\frac{x^3}{12}}dx=2+\frac{2^4}{12\cdot 4}+\frac{2^7}{12^2\cdot 2!\cdot 7}+\frac{2^{10}}{12^3\cdot 3!\cdot 10}+\dots$$

$$|R|=\frac{2^{13}}{12^2\cdot 4!\cdot 13}+\frac{2^{16}}{12^5\cdot 5!\cdot 16}+\dots<\frac{2^{13}}{12^4\cdot 4!\cdot 13}\cdot\left(1+\frac{2^3}{12\cdot 5}+\frac{2^6}{12^2\cdot 5^2}+\dots\right)=$$

$$=\frac{2^{13}}{12^4\cdot 4!\cdot 13}\cdot\frac{1}{1-\frac{2^3}{60}}=\frac{2^2\cdot 60}{3^5\cdot 13\cdot(60-2^3)}<0{,}0002$$

	$2=2{,}000$		
+	$\frac{24}{12\cdot 4}=0{,}3333$	(+)	с недостатком
	$\frac{2^7}{12^2\cdot 2!\cdot 7}=0{,}0635$	(-)	с избытком
	$\frac{2^{10}}{12^3\cdot 3!\cdot 10}=0{,}0099$	(-)	с избытком
	2,4067		

$$2{,}4066<\int_0^2 e^{\frac{x^3}{12}}dx<2{,}4070$$

<u>Ответ:</u> $\int_0^2 e^{\frac{x^3}{12}}dx=2{,}407$

4. $y'=\sin x+\cos y \qquad y|_{x=\pi/2}=\frac{\pi}{2}$

$$y(x)=y\left(\frac{\pi}{2}\right)+y'\left(\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right)+\frac{y''\left(\frac{\pi}{2}\right)}{2!}\cdot\left(x-\frac{\pi}{2}\right)^2+\frac{y'''\left(\frac{\pi}{2}\right)}{3!}\cdot\left(x-\frac{\pi}{2}\right)^3+\dots$$

$$y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$$

$$y'=\sin x+\cos y \qquad y'\left(\frac{\pi}{2}\right)=1$$

$$y'' = \cos x - \sin y y' \qquad y''\left(\frac{\pi}{2}\right) = -1$$

$$y''' = -\sin x - \cos y \cdot (y')^2 - y'' \sin y \qquad y'''\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} + \frac{\pi}{2} = 0$$

$$y(x) = \frac{\pi}{2} + \frac{1}{1!}\cdot\left(x - \frac{\pi}{2}\right) - \frac{1}{2!}\cdot\left(x - \frac{\pi}{2}\right)^2 + \ldots$$

5. $(x^2-3)y'' - 2xy' + 2y = (5 - x^2)\sin x - 2x\cos x \qquad y(0)=0 \qquad y'(0)=1$

$$\begin{array}{r|l} 2 & y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n + \cdots \\ -2 & xy'(x) = a_1x + 2a_2x^2 + 3a_3x^3 + \ldots + na_n\cdot x^n + \cdots \\ -3 & y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + 5\cdot 4a_5x^3 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots \\ 1 & x^2y''(x) = 2a_2x^2 + 3\cdot 2a_3x^3 + \cdots + n(n-1)a_nx^n + \cdots \end{array}$$

$$(2a_0 - 6a_2) - 3\cdot 3\cdot 2\cdot a_3x - 3\cdot 4\cdot 3a_4x^2 + (2a_3 - 3\cdot 5\cdot 4a_5)x^3 + \ldots + [(n-1)(n-2)a_n - 3(n+2)(n+1)a_{n+2}]x^n + \ldots$$

$$\begin{array}{r|l} 5 & \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^{n-1}\cdot\frac{x^{2n-1}}{(2n-1)!} + \ldots \\ - & x^2\sin x = x^3 - \frac{x^5}{3!} + \ldots + (-1)^n\cdot\frac{x^{2n-1}}{(2n-3)!} + \ldots \\ -2 & x\cos x = x - \frac{x^3}{2!} + \frac{x^5}{4!} - \ldots + (-1)^{n-1}\cdot\frac{x^{2n-1}}{(2n-2)!} + \ldots \end{array}$$

$$3x - \frac{5}{3!}x^3 + \frac{3}{4!}x^5 - \ldots + (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}x^{2n-1} + \ldots$$

$a_0 = 0$

$2a_0 - 6a_2 = 0$ $\qquad a_1 = 1$, $a_2 = 0$

$-3\cdot 3\cdot 2a_3 = 3$ $\qquad a_3 = -\frac{1}{3!}$

$-3\cdot 4\cdot 3a_4 = 0$ $\qquad a_4 = 0$

$(n-1)(n-2)\cdot a_n + (n+2)(n+1)\cdot a_{n+2} = 0 \qquad n = 2m$ - четные $\qquad a_{2n} = 0$

$$(2n-2)(2n-3)\cdot a_{2n-1} + (2n+1)\cdot 2n\cdot a_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}$$

Пусть $a_{2n-1} = (-1)^{n-1}\cdot\frac{1}{(2n-1)!}$

$$(2n-2)(2n-3)\cdot(-1)^{n-1}\cdot\frac{1}{(2n-1)!} - 3\cdot(2n+1)2na_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}$$

$$a_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9 - 4n^2 + 10n - 6}{(-3)\cdot(2n-1)!}$$

$$a_{2n+1} = (-1)^n\cdot\frac{1}{(2n-1)!}$$

$$y(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots + (-1)^n\cdot\frac{1}{(2n+1)!}x^{2n+1} + \ldots$$

$$y\left(\frac{3}{4}\right)=\frac{3}{4}-\frac{1}{3!}\cdot\left(\frac{3}{4}\right)^3+\frac{1}{5!}\cdot\left(\frac{3}{4}\right)^5-\ldots+(-1)^n\cdot\frac{1}{(2n+1)!}\cdot\left(\frac{3}{4}\right)^{2n+1}+\ldots$$

$$|R|<\frac{1}{5!}\cdot\left(\frac{3}{4}\right)^5<0{,}002$$

$$\frac{3}{4}=0{,}750$$

$$-\frac{1}{3!}\cdot\left(\frac{3}{4}\right)^3=0{,}070 \quad (+) \qquad \text{с недостатком}$$

$$0{,}680$$

$$y\left(\frac{3}{4}\right)\approx 0{,}68$$

Ответ: $y(x)=\sin x \qquad y\left(\frac{3}{4}\right)\approx 0{,}68$

6. $f(x)=\begin{cases}1 & -\pi<x<\frac{\pi}{2}\\ -1 & \frac{\pi}{2}<x<\pi\end{cases}$

$$a_0=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}dx-\int_{\pi/2}^{\pi}dx\right]=1$$

$$a_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\cos nxdx+\int_{\pi/2}^{\pi}-\cos nxdx\right]=\frac{1}{\pi}\cdot\left[\frac{1}{n}\cdot\sin nx\Big|_{-\pi}^{\pi/2}-\frac{1}{n}\cdot\sin nx\Big|_{\pi/2}^{\pi}\right]=\frac{2}{\pi n}\cdot\sin\frac{\pi n}{2}$$

$$b_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\sin nxdx-\int_{\pi/2}^{\pi}\sin nxdx\right]=\frac{1}{\pi}\cdot\left[-\frac{1}{n}\cos nx\Big|_{-\pi}^{\pi/2}+\frac{1}{n}\cos nx\Big|_{\pi/2}^{\pi}\right]=$$

$$=\frac{2}{\pi n}\cdot\left[(-1)^n-\cos\frac{n\pi}{2}\right]$$

Ответ: $f(x)=\frac{1}{2}+\frac{2}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{n\pi}{2}\cdot\cos nx+\frac{1}{n}\cdot\left[(-1)^n-\cos\frac{n\pi}{2}\right]\cdot\sin nx$

7. $f(x)=\begin{cases}1+x & 0<x<1\\ 3-x & 1<x<3\end{cases}$ по косинусам

$$a_0=\frac{2}{3}\cdot\left[\int_0^1(1+x)dx+\int_1^3(3-x)dx\right]=\frac{7}{3}$$

$$a_n=\frac{2}{3}\cdot\left[\int_0^1(1+x)\cdot\cos\frac{\pi nx}{3}dx+\int_1^3(3-x)\cos\frac{\pi nx}{3}dx\right]=$$

$$=\frac{2}{3}\cdot\left[\frac{3}{n\pi}\sin\frac{\pi nx}{3}+\frac{3x}{\pi n}\cdot\sin\frac{\pi nx}{3}+\frac{9}{n^2\pi^2}\cos\frac{\pi nx}{3}\right]\Bigg|_0^1+$$

$$+\left[\frac{9}{n\pi}\sin\frac{\pi nx}{3}-\frac{3x}{\pi n}\cdot\sin\frac{\pi nx}{3}-\frac{9}{n^2\pi^2}\cdot\cos\frac{\pi nx}{3}\right]\Bigg|_1^3=\frac{6}{n^2\pi^2}\cdot\left[2\cos\frac{n\pi}{3}-1+(-1)^{n+1}\right]$$

Ответ: $f(x)=\frac{7}{6}+\frac{6}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left[2\cos\frac{\pi n}{3}-1+(-1)^{n+1}\right]\cdot\cos\frac{\pi nx}{3}.$

Вариант 11.

1. $\sum_{n=1}^{\infty}\frac{4^n}{n^4+4^{-n}}\cdot\left(x+\frac{3}{2}\right)^{2n-1}$

$$\lim_{n\to\infty}\frac{4^{n+1}\cdot\left|x+\frac{3}{2}\right|^{2n+1}\cdot(n^4+4^{-n})}{[(n+1)^4+4^{-(n+1)}]\cdot\left|x+\frac{3}{2}\right|^{2n-1}\cdot 4^n}=4\cdot\left|x+\frac{3}{2}\right|^2; \qquad \left|x+\frac{3}{2}\right|<\frac{1}{2}$$

$x+\frac{3}{2}=\frac{1}{2}$ $\quad \sum_{n=1}^{\infty}\frac{2}{n^4+4^{-n}}$ сходится

$x+\frac{3}{2}=-\frac{1}{2}$ $\quad \sum_{n=1}^{\infty}\frac{(-1)^n\cdot 2}{n^4+4^{-n}}$ сходится

Ответ: $-2\le x\le -1$

2. $\cos^2 x$ $\qquad a=\frac{\pi}{3}$

$\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$

$$\cos 2x=\cos 2\cdot\left[\left(x-\frac{\pi}{3}\right)+\frac{\pi}{3}\right]=\cos\left[2\cdot\left(x-\frac{\pi}{3}\right)+\frac{2\pi}{3}\right]=\cos\frac{2\pi}{3}\cdot\cos 2\cdot\left(x-\frac{\pi}{3}\right)-$$

$$-\sin\frac{2\pi}{3}\cdot\sin 2\cdot\left(x-\frac{\pi}{3}\right)=-\frac{1}{2}\cos 2\cdot\left(x-\frac{\pi}{3}\right)-\frac{\sqrt{3}}{2}\sin 2\cdot\left(x-\frac{\pi}{3}\right)$$

Ответ:

$$\cos^2 x=\frac{1}{2}-\frac{1}{4}\cdot\sum_{n=0}^{\infty}(-1)^n\cdot\left[\frac{2^{2n}\cdot\left(x-\frac{\pi}{3}\right)^{2n}}{(2n)!}+(?)+\frac{2^{2n+1}\cdot\left(x-\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}\right],\quad -\infty<x<\infty$$

3. $\int_0^{0.5}\frac{dx}{\sqrt[5]{1+x^3}}$

$$\frac{1}{\sqrt[5]{1+x^3}}=(1+x^3)^{-\frac{1}{5}}=1-\frac{1}{5}x^3+\frac{1\cdot 6}{5^2\cdot 2!}x^6-\frac{1\cdot 6\cdot 11}{5^3\cdot 3!}x^9+\ldots$$

$$\int_0^{0.5}\frac{dx}{\sqrt[5]{1+x^3}}=\frac{1}{2}-\frac{1}{5\cdot 4\cdot 2^4}+\frac{1\cdot 6}{5^2\cdot 2!\cdot 7\cdot 2^7}-\frac{1\cdot 6\cdot 11}{5^3\cdot 3!\cdot 10\cdot 2^{10}}+\ldots$$

$|R|<\frac{1\cdot 6}{5^2\cdot 2!\cdot 7\cdot 2^7}<0{,}0002$ $\qquad \int_0^{0.5}\frac{1}{\sqrt[5]{1+x^3}}\approx\frac{1}{2}-\frac{1}{5\cdot 4\cdot 2^4}\approx 0{,}497$

4. $y'=\sin 2x+\cos y$ $\qquad y\left(\frac{\pi}{2}\right)=\pi$

$$y(x)=y\left(\frac{\pi}{2}\right)+\frac{y'\left(\frac{\pi}{2}\right)}{1!}\cdot\left(x-\frac{\pi}{2}\right)+\frac{y''\left(\frac{\pi}{2}\right)}{2!}\cdot\left(x-\frac{\pi}{2}\right)^2+\frac{y'''\left(\frac{\pi}{2}\right)}{3!}\cdot\left(x-\frac{\pi}{2}\right)^3+\ldots$$

$$y\left(\frac{\pi}{2}\right)=\pi$$

$y'=\sin 2x+\cos y$ $\qquad$ $y'\left(\frac{\pi}{2}\right)=\sin\pi+\cos\pi=-1$

$y''=2\cos 2x-y'\cdot\sin y$ $\qquad$ $y''\left(\frac{\pi}{2}\right)=2\cos\pi=-2$

$y'''=-4\sin 2x-y''\sin y-(y')^2\cos y$ $\qquad$ $y'''\left(\frac{\pi}{2}\right)=-4\sin\pi+\sin\pi-\cos\pi=1$

Ответ: $\quad y(x)=\pi-\frac{1}{1!}\cdot\left(x-\frac{\pi}{2}\right)-\frac{2}{2!}\cdot\left(x-\frac{\pi}{2}\right)^2+\frac{1}{3!}\cdot\left(x-\frac{\pi}{2}\right)^3+\ldots$

5. $\quad (5x-25)y''-xy'+y=10x-50-x^2 \qquad y(0)=1 \qquad y'(0)=\frac{1}{5}$

$$\begin{array}{r|l}
1 & y(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+\cdots \\
- & xy'(x)=a_1x+2a_2x^2+3a_3x^3+\ldots+na_n\cdot x^n+\cdots \\
-25 & y''(x)=2a_2+3\cdot 2a_3x+4\cdot 3a_4x^2+5\cdot 4a_5x^3+\cdots+(n+2)(n+1)a_{n+2}x^n+\cdots \\
5 & xy''(x)=2a_2x+3\cdot 2a_3x^2+4\cdot 3a_4x^3+\cdots+(n+1)n\cdot a_{n+1}x^n+\cdots \\
\hline
\end{array}$$

$$(a_0-50a_2)+(-25\cdot 3\cdot 2\cdot a_3+10a_2)\cdot x+(-a_2-25\cdot 4\cdot 3a_4+5\cdot 3\cdot 2a_3)x^2+$$
$$+(-2a_3-25\cdot 5\cdot 4a_5+5\cdot 4\cdot 3a_4)x^3+\ldots+[-(n-1)a_n-25(n+2)(n+1)a_{n+2}+$$
$$+5n(n+1)a_{n+1}]x^n+\ldots=10x-50-x^2$$

$a_0=1 \qquad a_1=\frac{1}{5}$

$a_0-50a_2=-50 \qquad a_2=\frac{51}{50}$

$-25\cdot 3\cdot 2a_3+10a_2=10 \qquad -25\cdot 3\cdot 2a_3+\frac{51}{50}\cdot 10=10 \qquad a_3=\frac{1}{5^3\cdot 3!}$

$[-(n-1)a_n-25\cdot(n+2)(n+1)a_{n+2}+5n(n+1)a_{n+1}]=0$

Пусть $\quad a_n=\frac{1}{5^n\cdot n!}, \quad a_{n+1}=\frac{1}{5^{n+1}(n+1)!}$

$$-25(n+2)(n+1)a_{n+2}=-\frac{5n(n+1)}{5^{n+1}(n+1)!}+\frac{(n-1)}{5^n\cdot n!}$$

$$-25(n+1)(n+2)a_{n+2}=\frac{-n+n-1}{5^n\cdot n!} \qquad a_{n+2}=\frac{1}{5^{n+2}(n+2)!}$$

$$y(x)=1+\frac{x}{5}+\frac{51}{5^2\cdot 2!}x^2+\frac{1}{5^3\cdot 3!}x^3+\ldots+\frac{x^n}{5^n\cdot n!}+\ldots$$

$$y(1)=1+\frac{1}{5}+\frac{51}{5^2\cdot 2!}+\frac{1}{5^3\cdot 3!}+\ldots+\frac{1}{5^n\cdot n!}+\ldots$$

$$|R|=\frac{1}{5^3\cdot 3!}+\frac{1}{5^4\cdot 4!}+\ldots<\frac{1}{5^3\cdot 3!}\left(1+\frac{1}{5\cdot 4}+\frac{1}{5^2\cdot 4^2}+\ldots\right)=\frac{1}{5^3\cdot 3!}\cdot\frac{1}{1-\frac{1}{20}}\le 0{,}002$$

$$y(1)\approx 1+\frac{1}{5}+\frac{51}{5^2\cdot 2!}=1+0{,}2+\frac{102}{100}=1+0{,}2+1{,}02=2{,}22$$

Ответ: $y(x)=\frac{x}{5}+x^2+\left(e^{x/5}-\frac{x}{5}\right)$

$y(1)\approx 2{,}22$

6. $f(x)=\begin{cases}0 & -1<x<0\\ x^2+1 & 0<x<1\end{cases}$

$$a_0=\int_0^1(x^2+1)dx=\frac{4}{3}$$

$$a_n=\int_0^1(x^2+1)\cos n\pi x dx=\frac{x^2}{\pi n}\sin n\pi x\Big|_0^1+\frac{2x}{n^2\pi^2}\cdot\cos n\pi x\Big|_0^1-$$

$$-\frac{2}{n^3\pi^3}\sin n\pi x\Big|_0^1+\frac{1}{n\pi}\cdot\sin n\pi x\Big|_0^1=(-1)^n\frac{2}{n^2\pi^2}$$

$$b_n=\int_0^1(x^2+1)\sin n\pi x dx=-\frac{x^2}{n\pi}\cos\pi nx\Big|_0^1+\frac{2x}{n^2\pi^2}\cdot\sin n\pi x\Big|_0^1+\frac{2}{n^3\pi^3}\cdot\cos n\pi x\Big|_0^1-$$

$$-\frac{x}{n\pi}\cos\pi nx\Big|_0^1=-\frac{2}{n\pi}\cdot(-1)^n+\frac{2}{n^3\pi^3}\cdot(-1)^n+\frac{1}{n\pi}-\frac{2}{n^3\pi^3}$$

$$f(x)=\frac{2}{3}+\sum_{n=1}^{\infty}\frac{2\cdot(-1)^n}{n^2\pi^2}\cdot\cos n\pi x+\left[\frac{1}{\pi n}-\frac{2\cdot(-1)^n}{\pi n}-\frac{2}{\pi^3 n^3}+\frac{2\cdot(-1)^n}{n^3\pi^3}\right]\cdot\sin\pi nx$$

7. $f(x)=\begin{cases}0 & 0<x<\frac{\pi}{4}\\ 1 & \frac{\pi}{4}<x<\frac{\pi}{2}\\ 0 & \frac{\pi}{2}<x<\pi\end{cases}$ по синусам

$$b_n=\frac{2}{\pi}\cdot\int_{\pi/4}^{\pi/2}\sin nx dx=-\frac{2}{\pi n}\cos nx\Big|_{\pi/4}^{\pi/2}=-\frac{2}{\pi n}\cdot\left[\cos\frac{n\pi}{2}-\cos\frac{n\pi}{4}\right]$$

Ответ: $f(x)=-\frac{2}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\left[\cos\frac{n\pi}{2}-\cos\frac{n\pi}{4}\right]\cdot\sin nx$

Вариант 12.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(x-3)^{2n}}{\sqrt{n+2}\cdot\sqrt[3]{n+3}}$

$$\lim_{n\to\infty}\frac{|x-3|^{2(n+1)}\cdot\sqrt{n+2}\cdot\sqrt[3]{n+3}}{\sqrt{n+3}\cdot\sqrt[3]{n+4}\cdot|x-3|^{2n}}=|x-3|^2;\qquad |x-3|<1$$

$x-3=\pm 1 \qquad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}\cdot\sqrt[3]{n+3}}$ сходится условно

Ответ: $2 \le x \le 4$

2. $\sin^2 x \qquad a=\frac{\pi}{4}$

$$\sin^2 x=\frac{1-\cos 2x}{2}$$

$$\cos 2x=\cos 2\cdot\left[\left(x-\frac{\pi}{4}\right)+\frac{\pi}{4}\right]=\cos\left[\frac{\pi}{2}+2\left(x-\frac{\pi}{4}\right)\right]=-\sin 2\left(x-\frac{\pi}{4}\right)$$

$$\sin^2 x=\frac{1}{2}+\frac{1}{2}\sin 2\cdot\left(x-\frac{\pi}{4}\right)=\frac{1}{2}+\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n+1}\cdot\left(x-\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}=$$

$-\infty<x<+\infty$

$$=\frac{1}{2}+\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n}\cdot\left(x-\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}$$

Ответ: $\sin^2 x=\frac{1}{2}+\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n}\cdot\left(x-\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}$

$-\infty<x<+\infty$

3. $\int_{-3/4}^{0}\frac{1}{x^2}\ln(1+x^4)dx$

$$\ln(1+x^4)=x^4-\frac{x^8}{2}+\frac{x^{12}}{3}-\dots+(-1)^{n-1}\cdot\frac{x^{4n}}{n}+\cdots$$

$$\frac{1}{x^2}\ln(1+x^4)=x^2-\frac{x^6}{2}+\frac{x^{10}}{3}-\dots+(-1)^{n-1}\cdot\frac{x^{4n-2}}{n}+\cdots$$

$$\int_{-3/4}^{0}\frac{1}{x^2}\ln(1+x^4)dx=\frac{1}{3}\cdot\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^7\cdot\frac{1}{2\cdot 7}+\left(\frac{3}{4}\right)^{11}\cdot\frac{1}{3\cdot 11}-\ldots+$$

$$+(-1)^{n-1}\cdot\left(\frac{3}{4}\right)^{4n-1}\frac{1}{n(4n-1)}+\ldots$$

$$|R|<\left(\frac{3}{4}\right)^{15}\cdot\frac{1}{4\cdot 15}<0{,}0003$$

$$\int_{-3/4}^{0}\frac{1}{x^2}\ln(1+x^4)dx\cong\left(\frac{3}{4}\right)^3\cdot\frac{1}{3}-\left(\frac{3}{4}\right)^7\cdot\frac{1}{2\cdot 7}+\left(\frac{3}{4}\right)^{11}\cdot\frac{1}{3\cdot 11}$$

$\left(\frac{3}{4}\right)^3\cdot\frac{1}{3}\approx 0{,}1406 \qquad (+) \qquad$ с недостатком

$$\left(\frac{3}{4}\right)^7 \cdot \frac{1}{2\cdot 7} \approx 0{,}0097 \qquad (+) \qquad \text{с недостатком}$$

$$\left(\frac{3}{4}\right)^{11} \cdot \frac{1}{3\cdot 11} \approx 0{,}0002 \qquad (+) \qquad \text{с недостатком}$$

0,1315

Ответ: $\int\limits_{-3/4}^{0} \frac{1}{x^2}\ln(1+x^4)dx \approx 0{,}132$

$$[-(n-1)a_n - 25(n+2)(n+1)a_{n+2} + 5n(n+1)a_{n+1}] = 0$$

Пусть $a_n = \frac{1}{5^n \cdot n!} \qquad a_{n+1} = \frac{1}{5^{n+1}(n+1)!}$

Тогда $-25(n+2)(n+1)a_{n+2} = (n-1)\frac{1}{5^n \cdot n!} - \frac{n}{5^n \cdot n!}$

$$a_{n+2} = \frac{1}{5^{n+2}(n+2)!}$$

$$y(x) = 1 + \frac{1}{5}x + \frac{51}{5^2 \cdot 2!}x^2 + \frac{751}{5^3 \cdot 3!}x^3 + \frac{1}{5^4 \cdot 4!}x^4 + \ldots + \frac{1}{5^n \cdot n!}x^n + \ldots$$

$$y(2) = 1 + \frac{1}{5}\cdot 2 + \frac{1}{5^2 \cdot 2!}\cdot 2^2 + \frac{751}{5^3 \cdot 3!}\cdot 2^3 + \frac{1}{5^4 \cdot 4!}\cdot 2^4 + \ldots +$$

$$|R| = \frac{1}{5^4 \cdot 4!}\cdot 2^4 + \frac{1}{5^5 \cdot 5!}\cdot 2^5 + \ldots < \frac{1}{5^4 \cdot 4!}\cdot 2^4 \cdot \left[1 + \frac{2}{5\cdot 5} + \frac{2^2}{5^4} + \ldots\right] =$$

$$= \frac{2^4}{5^4 \cdot 4!}\cdot \frac{1}{1-\frac{2}{25}} = \frac{2^4}{5^4 \cdot 4!}\cdot \frac{25}{23} = \frac{2^4}{5^2 \cdot 4! \cdot 23} < 0{,}002$$

$$1 + \frac{2}{5} = 1{,}400$$

$$\frac{2^2}{5^2 \cdot 2!} = 0{,}080$$

$$\frac{751}{5^3 \cdot 3!} = 8{,}011$$

9,491

Ответ: $y(2) \approx 9{,}49$

4. $y' = x^2 + 2\ln y \qquad y(-2) = 1$

$$y(x) = y(-2) + \frac{y'(-2)}{1!}\cdot(x+2) + \frac{y''(-2)}{2!}\cdot(x+2)^2 + \ldots +$$

$y(-2) = 1$

$y' = x^2 + 2\ln y \qquad y'(-2) = 4$

$y'' = 2x + \frac{2}{y}\cdot y' \qquad y''(-2) = 4$

$y''' = 2 - \frac{2}{y^2}\cdot(y')^2 + \frac{2y''}{y} \qquad y'''(-2) = -22$

[illegible]ет: $y(x)=1-\frac{4}{1!}\cdot(x+2)+\frac{4}{2!}\cdot(x+2)^2-\frac{22}{3!}\cdot(x+2)^3+\ldots$

$(5x-25)y''-xy'+y=30x^2-150x-2x^3 \qquad y(0)=1, \qquad y'(0)=\frac{1}{5}$

$$\begin{array}{r|l} 1 & y(x)=a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_nx^n+\cdots \\ - & xy'(x)=a_1x+2a_2x^2+3a_3x^3+\ldots+na_n\cdot x^n+\cdots \\ -25 & y''(x)=2a_2+3\cdot 2a_3x+4\cdot 3a_4x^2+5\cdot 4a_5x^3+\cdots+(n+2)(n+1)a_{n+2}x^n+\cdots \\ 5 & xy''(x)=2a_2x+3\cdot 2a_3x^2+4\cdot 3a_4x^3+\cdots+(n+1)n\cdot a_{n+1}x^n+\cdots \end{array}$$

$$(a_0-50a_2)+(-25\cdot 3\cdot 2\cdot a_3+10a_2)\cdot x+(-a_2-25\cdot 4\cdot 3a_4+5\cdot 3\cdot 2a_3)x^2+$$
$$+(-2a_3-25\cdot 5\cdot 4a_5+5\cdot 4\cdot 3a_4)x^3+\ldots+[-(n-1)a_n-25(n+2)(n+1)a_{n+2}+$$
$$+5n(n+1)a_{n+1}]x^n+\ldots=30x^2-150x-2x^3$$

$a_0=1 \qquad a_1=\frac{1}{5}$

$a_0-50a_2=0 \qquad a_2=\frac{1}{50}=\frac{1}{5^2\cdot 2!}$

$-25\cdot 3\cdot 2a_3+10a_2=-150$

$-25\cdot 3\cdot 2a_3=-150-\frac{10}{50} \qquad a_3=\frac{751}{5^3\cdot 2!}$

$a_2-25\cdot 4\cdot 3a_4=30+\frac{1}{5^2\cdot 2!}-\frac{751\cdot 2}{5^2\cdot 2!}$

$-25\cdot 4\cdot 3a_4=-\frac{1}{5^2\cdot 2!}- \qquad a_4=\frac{1}{5^4\cdot 4!}$

$-2a_3-25\cdot 5\cdot 4a_5+5\cdot 4\cdot 3a_4=-2$

$-25\cdot 5\cdot 4a_5=-2+\frac{1502}{5^3\cdot 3!}-\frac{3}{5^3\cdot 3!}$

$-25\cdot 5\cdot 4a_5=-\frac{1}{5^3\cdot 3!}$

Ответ: $a_5=\frac{1}{5^5\cdot 5!}$

6. $f(x)=\begin{cases}1-x^2 & -1<x<0\\ 0 & 0<x<1\end{cases}$

$a_0=\int\limits_{-1}^{0}(1-x^2)dx=\frac{2}{3}$

$a_n=\int\limits_{-1}^{0}(1-x^2)\cos n\pi x dx=-\frac{x^2}{\pi n}\sin n\pi x\Big|_{-1}^{0}-\frac{2x}{n^2\pi^2}\cdot\cos n\pi x\Big|_{-1}^{0}+$

$+\frac{2}{n^2\pi^2}\sin n\pi x\Big|_{-1}^{0}+\frac{1}{n\pi}\cdot\sin n\pi x\Big|_{0}^{1}=-\frac{2}{n^2\pi^2}\cdot(-1)^n$

$b_n=\int\limits_{-1}^{0}(1-x^2)\sin n\pi x dx=\frac{x^2}{n\pi}\cos\pi nx\Big|_{-1}^{0}-\frac{2x}{n^2\pi^2}\cdot\sin n\pi x\Big|_{-1}^{0}-\frac{2}{n^3\pi^3}\cdot\cos n\pi x\Big|_{-1}^{0}-$

$$-\frac{1}{n\pi}\cos n\pi n\Big|_{-1}^{0}=\frac{2}{n^3\pi^3}\cdot[(-1)^n-1]-\frac{1}{n\pi}$$

Ответ:

$$f(x)=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{2\cdot(-1)^{n+1}}{n^2\pi^2}\cdot\cos n\pi x+\left\{\frac{2\cdot[(-1)^n-1]}{n^3\pi^3}-\frac{1}{n\pi}\right\}\cdot\sin\pi nx$$

7. $f(x)=\begin{cases}0 & 0<x<\frac{3}{4}\pi\\ 2 & \frac{3}{4}\pi<x<\pi\end{cases}$ по синусам

$$b_n=\frac{2}{\pi}\cdot\int_{3\pi/4}^{\pi}2\sin nxdx=-\frac{4}{\pi n}\cos nx\Big|_{3\pi/4}^{\pi}=\frac{4}{\pi n}\cdot\left[\cos\frac{3n\pi}{2}-(-1)^n\right]$$

Ответ: $f(x)=\frac{4}{\pi}+\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left[\cos\frac{3n\pi}{4}-(-1)^n\right]\cdot\sin nx$

Вариант 13.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(x+4)^{2n}}{\sqrt{n+2}\cdot\sqrt[4]{n+4}}$

$$\lim_{n\to\infty}\frac{(x+4)^{2n+2}\cdot\sqrt{n+2}\cdot\sqrt[4]{n+4}}{\sqrt{n+3}\cdot\sqrt[4]{n+5}\cdot(x+4)^{2n}}=(x+4)^2; \qquad |x+4|<1$$

$x+4=\pm1$ $\quad\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}\cdot\sqrt[4]{n+4}}$ сходится условно

Ответ: $-5\le x\le-3$

2. $\frac{1}{1-x^2}=\frac{1}{2}\cdot\frac{1}{(1+x)}+\frac{1}{2\cdot(1-x)}$ по степеням $x-2$

$$\frac{1}{1+x}=\frac{1}{3+(x-2)}=\frac{1}{3\cdot\left(1+\frac{x-2}{3}\right)}=\frac{1}{3}\cdot\left(1-\frac{x-2}{3}+\frac{(x-2)^2}{3^2}-\ldots+(-1)^n\cdot\frac{(x-2)^n}{3^n}+\ldots\right)$$

$$\frac{1}{1-x}=\frac{1}{-1-(x-2)}=-\frac{1}{1+(x-2)}=-\left[1-(x-2)+(x-2)^2+\ldots+(-1)^n\cdot(x-2)^n+\ldots\right]$$

$$\frac{1}{1-x^2}=\frac{1}{2}\cdot\sum_{n=0}^{\infty}(-1)^n\cdot\left(\frac{1}{3^{n+1}}-1\right)\cdot(x-2)^n$$

$-1<x-2<1;\qquad 1<x<3$

Ответ: $\frac{1}{1-x^2}=\frac{1}{2}\cdot\sum_{n=0}^{\infty}(-1)^n\cdot\left(\frac{1}{3^{n+1}}-1\right)\cdot(x-2)^n$

$1<x<3$

3. $\int_0^1 x^2 arctg\frac{x^2}{4}dx$

$$arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots+(-1)^n\cdot\frac{x^{2n+1}}{2n+1}+\cdots \qquad -1<x<1$$

$$x^2 arctg\frac{x^2}{4} = \frac{x^4}{4} - \frac{x^8}{4^3\cdot 3} + \frac{x^{12}}{4^5\cdot 5} - \dots + (-1)^n \cdot \frac{x^{4n+4}}{4^{2n+1}\cdot(2n-1)} + \dots$$

$$\int_0^1 x^2 arctg\frac{x^2}{4}dx = \frac{1}{4\cdot 5} - \frac{1}{4^3\cdot 3\cdot 9} + \frac{1}{4^5\cdot 5\cdot 13} - \dots + (-1)^n \cdot \frac{1}{4^{2n+1}\cdot(2n-1)(4n+5)} + \dots$$

$$|R| < \frac{1}{4^3\cdot 3\cdot 9} < 0{,}0006$$

$$\int_0^1 x^2 arctg\frac{x^2}{4}dx \approx \frac{1}{4\cdot 5} = 0{,}050$$

<u>**Ответ:**</u> $\int_0^1 x^2 arctg\frac{x^2}{4}dx = 0{,}050$

4. $y' = 2x - \ln y + 3$ $\qquad y(-3) = 1$

$$y(x) = y(-3) + \frac{y'(-3)}{1!}\cdot(x+3) + \frac{y''(-3)}{2!}\cdot(x+3)^2 + \frac{y'''(-3)}{3!}\cdot(x+3)^3 + \dots$$

$y' = 2x - \ln y + 3$ $\qquad y'(-3) = -6 + 3 = -3$

$y'' = 2 - \frac{y'}{y}$ $\qquad y''(-3) = 2 + 3 = 5$

$y''' = -\frac{y''y - (y')^2}{y^2}$ $\qquad y'''(-3) = -\frac{5-9}{1} = 4$

<u>**Ответ:**</u> $y(x) = 1 - 3\cdot(x+3) + \frac{5}{2!}\cdot(x+3)^2 + \frac{4}{3!}\cdot(x+3)^3 + \dots$

5. $(x+1)^2 y'' - 2(x+1)y' + 2y = 2\ln(1+x) - 3$ $\qquad y(0) = 1, \qquad y'(0) = 1$

2	$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots$
-2	$y'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots + (n+1)a_{n+1}\cdot x^n + \dots$
-2	$xy'(x) = a_1 x + 2a_2 x^2 + 3a_3 x^3 + \dots + na_n \cdot x^n + \dots$
1	$y''(x) = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3a_4 x^2 + 5\cdot 4a_5 x^3 + \dots + (n+2)(n+1)a_{n+2}x^n + \dots$
2	$xy''(x) = 2a_2 x + 3\cdot 2a_3 x^2 + 4\cdot 3a_4 x^3 + \dots + (n+1)n\cdot a_{n+1}x^n + \dots$
1	$x^2 y''(x) = 2a_2 x^2 + 3\cdot 2a_3 x^3 + \dots + n(n-1)\cdot a_n x^n + \dots$

$$2(a_0 - a_1 + a_2) + 3\cdot 2\cdot a_3 x + (3\cdot 2a_3 + 4\cdot 3a_4)x^2 + (2a_3 + 16a_4 + 5\cdot 4a_5)x^3 + \dots +$$
$$+ [(n-1)(n-2)a_n + 2(n^2-1)a_{n+1} + (n+2)(n+1)a_{n+2}]x^n + \dots$$

$$2\ln(1+x) - 3 = 2\cdot\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1}\cdot\frac{x^n}{n} + \dots\right) - 3 =$$

$$= -3 + 2x - x^2 + \frac{2}{3}\cdot x^3 - \dots + (-1)^{n-1}\cdot\frac{2}{n}x^n + \dots$$

$a_0 = 0$ $\qquad a_1 = 1$

$2(a_0 - a_1 + a_2) = -3$ $\qquad -2 + 2a_2 = -3$ $\qquad a_2 = -\frac{1}{2}$

$3\cdot 2a_3 = 2$ $\qquad a_3 = -\frac{1}{3}$

$2\cdot 3\cdot 2a_3 + 4\cdot 3a_4 = -1$ $\qquad a_4 = -\frac{1}{4}$

$$(n-1)(n-2)a_n + 2(n^2-1)a_{n+1} + (n+2)(n+1)a_{n+2} = (-1)^{n-1}\cdot\frac{2}{n}$$

Пусть $a_n = (-1)^{n-1}\cdot\frac{1}{n}$, $\quad a_{n+1} = (-1)^n\cdot\frac{1}{n+1}$

$$(-1)^{n-1}\cdot\frac{(n-1)(n-2)}{n} + (-1)^n\cdot 2\cdot(n-1) + (n+2)(n+1)a_{n+2} = (-1)^{n-1}\cdot\frac{2}{n}$$

$$(n+2)(n+1)a_{n+2} = (-1)^{n-1}\cdot\frac{n^2+n}{n} \qquad a_{n+2} = (-1)^{n+1}\cdot\frac{1}{n+2}$$

$$y(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots + (-1)^{n-1}\cdot\frac{1}{n}x^n + \ldots = \ln(1+x)$$

$$y\left(-\frac{1}{5}\right) = -\frac{1}{5} - \frac{1}{2\cdot 5^2} - \frac{1}{3\cdot 5^3} - \ldots - \frac{1}{n\cdot 5^n} - \ldots$$

$$|R| = \frac{1}{3\cdot 5^3} + \frac{1}{4\cdot 5^4} + \frac{1}{5\cdot 5^5} + \cdots < \frac{1}{3\cdot 5^3}\cdot\left(1 + \frac{1}{5} + \frac{1}{5^2} + \ldots\right) = \frac{1}{3\cdot 5^3\cdot\left(1-\frac{1}{5}\right)} =$$

$$= \frac{1}{3\cdot 5^3\cdot 4} = \frac{1}{300} < 0{,}007$$

$$y\left(-\frac{1}{5}\right) = -\frac{1}{5} - \frac{1}{2\cdot 5^2} = -\frac{1}{5} - \frac{1}{50} = -0{,}22$$

Ответ: $y(x) = \ln(1+x)$

$y\left(-\frac{1}{5}\right) = -0{,}22$

6. $f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ -1 & -\frac{\pi}{2} < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\cdot\int_{-\pi/2}^{\pi} -dx = -\frac{3}{2}$$

$$a_n = \frac{1}{\pi}\cdot\int_{-\pi/2}^{\pi} -\cos nx dx = -\frac{1}{n\pi}\sin nx\Big|_{-\pi/2}^{\pi} = -\frac{1}{\pi n}\cdot\sin\frac{\pi n}{2}$$

$$b_n = \frac{1}{\pi}\cdot\int_{-\pi/2}^{\pi} -\sin nx dx = \frac{1}{n\pi}\cdot\cos nx\Big|_{-\pi/2}^{\pi} = \frac{1}{n\pi}\cdot\left[(-1)^n - \cos\frac{n\pi}{2}\right]$$

Ответ: $f(x) = -\frac{3}{4} + \frac{1}{\pi}\cdot\sum_{n=1}^{\infty} -\frac{1}{n}\sin\frac{n\pi}{2}\cos nx + \frac{1}{n}\cdot\left[(-1)^n - \cos\frac{n\pi}{2}\right]\cdot\sin nx$

7. $f(x) = \begin{cases} 4-x^2 & 0 < x < 1 \\ 0 & 1 < x < 4 \end{cases}$ по косинусам

$$a_0 = \frac{2}{4}\cdot\int_0^1 (4-x^2)dx = \frac{11}{16}$$

$$a_n = \frac{2}{4} \cdot \int_0^1 (4 - x^2) \cos\frac{n\pi x}{4} dx = \frac{1}{2}\left[\frac{16}{n\pi}\sin\frac{n\pi x}{4} - \frac{4x^2}{n\pi}\sin\frac{n\pi x}{4} - \frac{32}{n^2\pi^2}\cos\frac{n\pi x}{4} + \right.$$

$$\left. + \frac{128}{n^3\pi^3}\sin\frac{n\pi x}{4}\right]\Bigg|_0^1 = \frac{2}{\pi}\left[\frac{3}{n}\sin\frac{n\pi}{4} + \frac{32}{n^3\pi^2}\sin\frac{n\pi}{4} - \frac{8}{n^2\pi}\cos\frac{n\pi}{4}\right]$$

Ответ: $f(x) = \frac{11}{12} + \frac{2}{\pi} \cdot \sum_{n=1}^{\infty}\left[\frac{3}{n}\sin\frac{n\pi}{4} + \frac{32}{n^3\pi^2}\sin\frac{n\pi}{4} - \frac{8}{n^2\pi}\cos\frac{\pi n}{4}\right] \cdot \cos\frac{\pi n x}{4}$

Вариант 14.

1. $\sum_{n=1}^{\infty}\frac{n(x+3)^n}{(n+2)\cdot 3^{2n}}$

$$\lim_{n\to\infty}\frac{(n+1)|x+3|^{n+1}(n+2)\cdot 3^{2n}}{(n+3)\cdot 3^{2(n+1)}\cdot n|x+3|^n} = \frac{|x+3|}{9}; \qquad |x+3| < 9$$

$x + 3 = 9$ $\quad \sum_{n=1}^{\infty}\frac{n}{n+2}$ - расходится

$x + 3 = -9$ $\quad \sum_{n=1}^{\infty}\frac{(-1)^n \cdot n}{n+2}$ - расходится

Ответ: $-12 < x < 6$

2. $\ln(1-x)$ по степеням $x+1$

$$\ln(1-x) = \ln[2-(x+1)] = \ln 2 + \ln\left[1 - \frac{x+1}{2}\right] = \ln 2 - \frac{x+1}{2} - \frac{(x+1)^2}{2^2 \cdot 2} -$$

$$- \frac{(x+1)^3}{2^3 \cdot 3} - \ldots - \frac{(x+1)^n}{2^n \cdot n} - \ldots$$

$\left|\frac{x+1}{2}\right| < 1 \qquad -3 \le x < 1$

Ответ: $\ln(1-x) = \ln 2 - \sum_{n=1}^{\infty}\frac{(x+1)^n}{2^n \cdot n}$

$-3 \le x < 1$

3. $\int_0^5 \frac{1}{x} arctg\frac{x}{10} dx$

$$arctg\frac{x}{10} = \frac{x}{10} - \frac{x^3}{10^3 \cdot 3} + \frac{x^5}{10^5 \cdot 5} - \cdots + (-1)^n \cdot \frac{x^{2n+1}}{10^{2n+1}(2n+1)} + \cdots \qquad -10 < x < 10$$

$$\frac{1}{x} arctg\frac{x}{10} = \frac{1}{10} - \frac{x^2}{10^3 \cdot 3} + \frac{x^4}{10^5 \cdot 5} - \cdots + (-1)^n \cdot \frac{x^{2n}}{10^{2n+1} \cdot (2n+1)} + \cdots$$

$$\int_0^5 \frac{1}{x} arctg\frac{x}{10} dx = \frac{5}{10} - \frac{5^3}{10^3 \cdot 3^2} + \frac{5^5}{10^5 \cdot 5^2} - \cdots + (-1)^n \cdot \frac{5^{2n+1}}{10^{2n+1} \cdot (2n-1)^2} + \cdots$$

$$|R| < \frac{5^7}{10^7 \cdot 7^2} < 0{,}0002$$

$$\int_0^5 \frac{1}{x} arctg\frac{x}{10}\,dx \approx \frac{5}{10} - \frac{5^3}{10^3\cdot 3^2} + \frac{5^5}{10^5\cdot 5^2}$$

+	$\frac{5}{10} = 0{,}500$		
-	$\frac{5^3}{10^3\cdot 3^2} = 0{,}014$	(-)	с избытком
+	$\frac{5^5}{10^5\cdot 5^2} = 0{,}001$	(+)	с недостатком
	0,487		

Ответ: $\int_0^5 \frac{1}{x} arctg\frac{x}{10}\,dx \approx 0{,}487$

4. $y' = 2\ln y - xy$ $\qquad\qquad y(2) = 1$

$$y(x) = y(2) + \frac{y'(2)}{1!}\cdot(x-2) + \frac{y''(2)}{2!}\cdot(x-2)^2 + \frac{y'''(2)}{3!}\cdot(x-2)^3 + \ldots$$

$y(2) = 1$

$y' = 2\ln y - xy$ $\qquad\qquad y'(2) = -2$

$y'' = \frac{2y'}{y} - (y + xy')$ $\qquad\qquad y''(2) = -4 - (1-4) = -1$

$y''' = 2\cdot\frac{y''y - (y')^2}{y^2} - (2y' + xy'')$ $\qquad\qquad y'''(2) = 2\cdot\frac{-1-4}{1} - (-4-2) = -10 + 6 = -4$

Ответ: $y(x) = 1 - \frac{2}{1!}\cdot(x-2) - \frac{1}{2!}\cdot(x-2)^2 - \frac{4}{3!}\cdot(x-2)^3 + \ldots$

5. $(1+x)y'' + xy' - y = 8$ $\qquad\qquad y(0) = -7,\quad y'(0) = -1$

-1	$y(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$
1	$xy'(x) = a_1x + 2a_2x^2 + \ldots + na_n\cdot x^n + \cdots$
1	$y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots$
1	$xy''(x) = 2a_2x + 3\cdot 2a_3x^2 + \cdots + (n+1)n\cdot a_{n+1}x^n + \cdots$

$$(-a_0 + 2a_2) + (3\cdot 2\cdot a_3 + 2a_2)x + (a_2 + 4\cdot 3a_4 + 3\cdot 2a_3)x^2 + \ldots +$$
$$+ [(n-1)a_n + (n+2)(n+1)a_{n+2} + (n+1)na_{n+1}]x^n + \ldots = 8$$

$a_0 = -7$ $\qquad\qquad a_1 = -1$

$-a_0 + 2a_2 = 8$ $\qquad\qquad a_2 = \frac{1}{2!}$

$3\cdot 2a_3 + 2a_2 = 0$ $\qquad\qquad a_3 = -\frac{1}{3!}$

$(n-1)a_n + (n+2)(n+1)a_{n+2} + (n+1)na_{n+1} = 0$

Пусть $a_n = (-1)^n\cdot\frac{1}{n!}$, $\qquad a_{n+1} = (-1)^{n+1}\cdot\frac{1}{(n+1)!}$

$(n+2)(n+1)a_{n+2} = (-1)^{n+1}\cdot(n-1)\cdot\frac{1}{n} + (-1)^n\cdot\frac{n}{n!}$

$$a_{n+2}=(-1)^n\cdot\frac{1}{n!(n+1)(n+2)} \qquad a_{n+2}=\frac{(-1)^{n+2}}{(n+2)!}$$

$$y(x)=-7-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots(-1)^n\cdot\frac{x^n}{n!}+\ldots=-8+e^{-x}$$

$$y\left(-\frac{1}{3}\right)=-7+\frac{1}{3}+\frac{1}{3^2\cdot 2!}+\frac{1}{3^3\cdot 3!}+\cdots+\frac{1}{3^n\cdot n!}+\ldots$$

$$|R|=\frac{1}{3^4\cdot 4!}+\frac{1}{3^5\cdot 5!}+\frac{1}{3^6\cdot 6!}+\cdots<\frac{1}{3^4\cdot 4!}\cdot\left(1+\frac{1}{3\cdot 5}+\frac{1}{3^2\cdot 5^2}+\ldots\right)=$$

$$=\frac{1}{3^3\cdot 4!}\cdot\frac{1}{1-\frac{1}{15}}=\frac{15}{3^4\cdot 4!\cdot 14}=\frac{5}{3^4\cdot 16\cdot 7}<0{,}0006$$

$\frac{1}{3}=0{,}333$ с недостатком

$\frac{1}{3^2\cdot 2!}=0{,}056$ с недостатком

$\frac{1}{3^3\cdot 3!}=0{,}006$ с недостатком

0,395

$$y\left(-\frac{1}{3}\right)\approx -7+0{,}4=-6{,}60$$

Ответ: $y(x)=-8+e^{-x}$

$y\left(-\frac{1}{3}\right)=-6{,}60$

6. $f(x)=\begin{cases}-\frac{x}{\pi} & -\pi<x<0\\ 0 & 0<x<\pi\end{cases}$

$$a_0=\frac{1}{\pi}\cdot\int_{-\pi}^{0}-\frac{x}{\pi}dx=\frac{1}{2}$$

$$a_n=\frac{1}{\pi}\cdot\int_{-\pi}^{0}-\frac{x}{\pi}\cos n\pi\, xdx=-\frac{1}{\pi^2}\left[\frac{x}{n}\sin nx+\frac{1}{n^2}\cos nx\right]_{-\pi}^{0}=\frac{1}{n^2\pi^2}\cdot[(-1)^n-1]$$

$$b_n=\frac{1}{\pi}\cdot\int_{-\pi}^{0}-\frac{x}{\pi}\sin nxdx=\frac{1}{\pi^2}\left[-\frac{x}{n}\cos nx+\frac{1}{n^2}\sin nx\right]_{-\pi}^{0}=\frac{1}{n\pi}(-1)^n$$

Ответ: $f(x)=\frac{1}{4}+\frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2\pi}\cdot[(-1)^n-1]\cos nx+\frac{1}{n}(-1)^n\sin nx$

7. $f(x)=\begin{cases}3x-3 & 0<x<1\\ 0 & 1<x<3\end{cases}$ по синусам

$$b_n=\frac{2}{3}\cdot\int_0^1(3x-3)\sin\frac{n\pi x}{3}dx=2\cdot\left[-\frac{3x}{n\pi}\cos\frac{n\pi x}{3}+\frac{9}{n^2\pi^2}\sin\frac{n\pi x}{3}+\right.$$

$$\left.+\frac{3}{n\pi}\cos\frac{n\pi x}{3}\right]_0^1=\frac{6}{n\pi}\left[\frac{3}{n\pi}\sin\frac{n\pi}{3}-1\right]$$

Ответ: $f(x)=\frac{6}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left[\frac{3}{n\pi}\sin\frac{n\pi}{3}-1\right]\cdot\sin\frac{\pi n x}{3}$

Вариант 15.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot 9^{n-\frac{1}{n}}\cdot(x+2)^{2n-1}$

$$\lim_{n\to\infty}\frac{|x+2|^{2n+1}\cdot 9^{n+1}\cdot 9^{-\frac{1}{n+1}}}{9^n\cdot 9^{-\frac{1}{n}}\cdot|x+2|^{2n-1}}=9|x+2|^2$$

$|x+2|<\frac{1}{3}$

$x+2=\frac{1}{3}$ $\sum_{n=1}^{\infty}(-1)^n\cdot 3^{\frac{n-2}{n}}$ расходится

$x+2=-\frac{1}{3}$ $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot 3^{\frac{n-2}{n}}$ расходится

Ответ: $-\frac{7}{3}<x<-\frac{5}{3}$

2. x^2e^x по степеням $x-1$

$x^2=(x-1)^2+2(x-1)+1$

$x^2e^x=[(x-1)^2+2(x-1)+1]\cdot e^{x-1}\cdot e$

$$1\quad e^{x-1}=1+(x-1)+\frac{(x-1)^2}{2!}+\ldots+\frac{(x-1)^n}{n!}+\ldots$$

$$1\quad 2(x-1)\cdot e^{x-1}=2(x-1)+2(x-1)^2+\ldots+\frac{2(x-1)^n}{(n-1)!}+\ldots$$

$$1\quad (x-1)^2e^{x-1}=(x-1)^2+\ldots+\frac{(x-1)^n}{(n-2)!}$$

$$1+3(x-1)+\frac{7}{2!}(x-1)^2+\ldots+\frac{n^2+n+1}{n!}(x-1)^n+\ldots \qquad -\infty<x<+\infty$$

Ответ: $x^2e^x=e\cdot\sum_{n=0}^{\infty}\frac{n^2+n+1}{n!}(x-1)^n$

$-\infty<x<+\infty$

3. $\int_0^3\frac{1}{x^2}\left(sh\frac{x}{3}-\sin\frac{x}{3}\right)dx$

$$sh\frac{x}{3}-\sin\frac{x}{3}=\frac{2x^3}{3^3\cdot 3!}+\frac{2x^7}{3^7\cdot 7!}+\cdots+\frac{2x^{4n-1}}{3^{4n-1}\cdot(4n-1)!}+\cdots$$

$$\frac{1}{x^2}\left(sh\frac{x}{3}-\sin\frac{x}{3}\right)dx=\frac{2x}{3^3\cdot 3!}+\frac{2x^5}{3^7\cdot 7!}+\cdots+\frac{2x^{4n-3}}{3^{4n-1}\cdot(4n-1)!}+\cdots$$

$-\infty<x<+\infty$

$$\int_0^3\frac{1}{x^2}\left(sh\frac{x}{3}-\sin\frac{x}{3}\right)dx=\frac{2\cdot 3^2}{2\cdot 3^3\cdot 3!}+\frac{2\cdot 3^6}{6\cdot 3^7\cdot 7!}+\cdots+\frac{2\cdot 3^{4n-2}}{(4n-2)\cdot 3^{4n-1}\cdot(4n-1)!}+\cdots$$

$$|R|=\frac{2\cdot 3^6}{6\cdot 3^7\cdot 7!}+\frac{2\cdot 3^{10}}{10\cdot 3^{11}\cdot 11!}+\cdots<\frac{2\cdot 3^6}{6\cdot 3^7\cdot 7!}\cdot\left(1+\frac{1}{8\cdot 9\cdot 10\cdot 11}+\frac{1}{(8\cdot 9\cdot 10\cdot 11)^2}+\cdots\right)=$$

$$=\frac{1}{3^2\cdot 7!}\cdot\frac{1}{1-\frac{1}{8\cdot 9\cdot 10\cdot 11}}=\frac{8\cdot 9\cdot 10\cdot 11}{3^2\cdot 7!(8\cdot 9\cdot 10\cdot 11-1)}<0{,}00003$$

$$\int_0^3\frac{1}{x^2}\left(sh\frac{x}{3}-\sin\frac{x}{3}\right)dx\approx\frac{2\cdot 3^2}{2\cdot 3^2\cdot 3!}=\frac{1}{3\cdot 3!}=\frac{1}{18}\approx 0{,}056$$

Ответ: $\int_0^3\frac{1}{x^2}\left(sh\frac{x}{3}-\sin\frac{x}{3}\right)dx\approx 0{,}056$

4. $y'=x-y+3e^y$ $\qquad y(-5)=0$

$$y(x)=y(-5)+\frac{y'(-5)}{1!}\cdot(x+5)+\frac{y''(-5)}{2!}\cdot(x+5)^2+\frac{y'''(-5)}{3!}\cdot(x+5)^3+\dots$$

$y(-5)=0$

$y'=x-y+3e^y$ $\qquad y'(-5)=-5+3=-2$

$y''=1-y'+3e\cdot y'$ $\qquad y''(-5)=1+2-6=-3$

$y'''=-y''+3\cdot\frac{e^y}{1}[(y')^2+y'']$ $\qquad y'''(-5)=3+3\cdot(4-3)=6$

Ответ: $y(x)=-2(x+5)-\frac{3}{2!}\cdot(x+5)^2+\frac{6}{3!}\cdot(x+5)^3+\dots$

5. $(x^2+1)y''+4xy'+2y=2\ln(1+x^2)+\frac{2+6x^2}{1+x^2}$ $\qquad y(0)=0,\qquad y'(0)=0$

$$\begin{array}{c|l}
2 & y(x)=a_0+a_1x+a_2x^2+\dots+a_nx^n+\cdots\\
4 & xy'=a_1x+2a_2x^2+\dots+na_n\cdot x^n+\cdots\\
1 & y''=2a_2+3\cdot 2a_3x+4\cdot 3a_4x^2+\dots+(n+2)(n+1)a_{n+2}x^n+\cdots\\
1 & x^2y''=2a_2x^2+\dots+n(n-1)a_nx^n+\cdots\\
\hline
\end{array}$$

$$(2a_0+2a_2)+(6a_1+3\cdot 2a_3)x+(12a_2+4\cdot 3a_4)x^2+\dots+$$
$$+[(n+2)(n+1)a_{n+2}+(n+2)(n+1)a_n]\cdot x^n$$

$$\begin{array}{c|ll}
2 & \ln(1+x^2)=x^2-\frac{x^2\cdot x^2}{2}+\frac{x^6}{3}-\dots+(-1)^{n-1}\cdot\frac{x^{2n}}{n}+\dots & -1<x\le 1\\
2 & \frac{1}{1+x^2}=1-x^2+x^4-\dots+(-1)^n\cdot x^{2n}+\cdots & -1<x<1\\
6 & \frac{x^2}{1+x^2}=x^2-x^4+\dots+(-1)^{n-1}\cdot x^{2n}+\cdots & \\
\hline
\end{array}$$

$$2+6x^2-5x^4+\dots+2\cdot(-1)^{n-1}\cdot\frac{2n+1}{n}x^{2n}+\cdots$$

$a_0=0$ $\qquad a_1=0$

$2a_0+2a_2=2$ $\qquad a_2=1$

$6a_1+6a_3=0$ $\qquad a_3=0$ $\qquad a_{2n+1}=0$

$$12a_2 + 12a_4 = 6 \qquad a_4 = -\frac{1}{2}$$

$$30a_4 + 30a_6 = -5 \qquad a_4 = \frac{1}{3}$$

$$(2n+2)(2n+1)a_{2n+2} + (2n+2)(2n+1)a_{2n} = 2\cdot(-1)^{n-1}\cdot\frac{2n+1}{n}$$

Пусть $a_{2n} = (-1)^{n-1}\cdot\frac{1}{n}$

$$(2n+2)(2n+1)a_{2n+2} + (2n+2)(2n+1)\cdot(-1)^{n-1}\cdot\frac{1}{n} = 2\cdot(-1)^{n-1}\cdot\frac{2n+1}{n}$$

$$(n+1)a_{2n+2} = (-1)^n\cdot\frac{n+1}{n} + (-1)^{n-1}\cdot\frac{1}{n}$$

$$a_{2n+2} = (-1)^n\cdot\frac{1}{n+1}$$

$$y(x) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \ldots + (-1)^{n-1}\cdot\frac{x^{2n}}{n} + \ldots = \ln(1+x^2)$$

$$y\left(\frac{1}{2}\right) = \frac{1}{2^2} - \frac{1}{2^4\cdot 2} + \frac{1}{2^6\cdot 3} - \frac{1}{2^8\cdot 4} + \ldots$$

$$|R| < \frac{1}{2^6\cdot 3} < 0{,}006$$

+	$\frac{1}{2^2} = 0{,}250$
-	$\frac{1}{2^5} = 0{,}031$
	0,219

Ответ: $y(x) = \ln(1+x^2)$

$$y\left(\frac{1}{2}\right) = 0{,}22$$

6. $f(x) = \begin{cases} 0 & -\pi < x < \frac{\pi}{2} \\ x & \frac{\pi}{2} < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\cdot\int_{\pi/2}^{\pi} x\,dx = \frac{3}{8}\pi$$

$$a_n = \frac{1}{\pi}\int_{\pi/2}^{\pi} x\cos nx\,dx = \frac{1}{\pi}\left[\frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\right]_{\pi/2}^{\pi} = \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} + \frac{1}{n^2}\cos\frac{n\pi}{2} - \frac{\pi}{2n}\sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{1}{\pi}\int_{\pi/2}^{\pi} x\sin nx\,dx = \frac{1}{\pi}\left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_{\frac{\pi}{2}}^{\pi} = \frac{1}{\pi}\left[-\frac{\pi}{n}\cdot(-1)^n + \frac{\pi}{2n}\cos\frac{n\pi}{2} - \frac{1}{n^2}\sin\frac{n\pi}{2}\right]$$

Ответ: $f(x) = \frac{3}{16}\pi + \frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\cos\frac{n\pi}{2} - \frac{\pi}{2n}\sin\frac{n\pi}{2}\right]\cos nx +$

$$+\left[-\frac{\pi}{n}\cdot(-1)^n+\frac{\pi}{2n}\cos\frac{n\pi}{2}-\frac{1}{n^2}\sin\frac{n\pi}{2}\right]\sin nx$$

7. $f(x)=\begin{cases}3-3x & 0<x<1\\ 0 & 1<x<3\end{cases}$ по косинусам

$$a_0=\frac{2}{3}\cdot\int_0^1(3-3x)dx=1$$

$$a_n=\frac{2}{3}\cdot\int_0^1(3-3x)\cos\frac{n\pi x}{3}dx=2\cdot\left[\frac{3}{n\pi}\sin\frac{n\pi x}{3}-\frac{3x}{n\pi}\sin\frac{n\pi x}{3}-\right.$$

$$\left.-\frac{9}{n^2\pi^2}\cos\frac{n\pi x}{3}\right]_0^1=\frac{18}{n^2\pi^2}\left[1-\cos\frac{n\pi}{3}\right]$$

Ответ: $f(x)=\frac{1}{2}+\frac{18}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left[1-\cos\frac{n\pi}{3}\right]\cdot\cos\frac{\pi nx}{3}$

Вариант 16.

1. $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\ln(n+1)}$;

$$\lim_{n\to\infty}\frac{|x+2|^{n+1}\cdot\ln(n+1)}{\ln(n+2)\cdot|x+2|^n}=|x+2|<1$$

$x+2=1$ $\quad\sum_{n=1}^{\infty}\frac{1}{\ln(n+1)}$ - расходится $\left(\frac{1}{\ln(n+1)}>\frac{1}{n+1}\right)$

$x+2=-1$ $\quad\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln(n+1)}$ - сходится условно

Ответ: $-3\le x<-1$

2. $f(x)=x^2e^{-x}$ по степеням $x+1$

$$x^2e^{-x}=[(x+1)^2-2(x+1)+1]\cdot e^{-(x+1)+1}=e\cdot[(x+1)^2-2(x+1)+1]\cdot\sum_{n=0}^{\infty}\frac{(-1)^n\cdot(x+1)^n}{n!}=$$

$$=e\cdot\left[1-3\cdot(x-1)+\sum_{n=2}^{\infty}\left(\frac{(-1)^n}{n!}+\frac{2\cdot(-1)^n}{(n-1)!}+\frac{(-1)^n}{(n-2)!}\right)\cdot(x+1)^n\right]$$

$$-\infty<x<+\infty$$

Ответ: $f(x)=e\cdot\left[1-3\cdot(x-1)+\sum_{n=2}^{\infty}\left(\frac{(-1)^n}{n!}+\frac{2\cdot(-1)^n}{(n-1)!}+\frac{(-1)^n}{(n-2)!}\right)\cdot(x+1)^n\right]$

$$-\infty<x<+\infty$$

3. $\int_0^2\frac{1}{x}\left(sh\frac{x}{2}+\sin\frac{x}{2}\right)dx$

$shx=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$; $\quad\sin x=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{2n+1}}{(2n+1)!}$

$$\frac{1}{x}\cdot\left[sh\frac{x}{2}+\sin\frac{x}{2}\right]=2\sum_{n=0}^{\infty}\frac{x^{4n}}{(4n+1)!\cdot 2^{4n+1}}$$

$$\int_0^2\frac{1}{x}\left[sh\frac{x}{2}+\sin\frac{x}{2}\right]dx=2\sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n+2)!\cdot 2^{4n+1}\cdot(4n+1)}\Bigg|_0^2=$$

$$=2\sum_{n=0}^{\infty}\frac{1}{(4n+1)!(4n+1)}\approx 2\cdot\left(1+\frac{1}{5!\cdot 5}\right)\approx 2{,}003$$

$$R_n=2\cdot\left[\frac{1}{(4n+1)!\cdot(4n+1)}+\frac{1}{(4n+5)!\cdot(4n+5)}+\ldots\right]<\frac{2}{(4n+1)!\cdot(4n+1)}\times$$

$$\times\left[1+\frac{1}{(4n+1)^4}+\ldots\right]=\frac{2}{(4n+1)!\cdot(4n+1)}\cdot\frac{(4n+1)^4}{(4n+1)^4-1}=\frac{2(4n+1)^2}{(4n)!\cdot[(4n+1)^4-1]}$$

$n=2$ $\qquad R_n<0{,}000006$

Ответ: $\int_0^2\frac{1}{x}\left(sh\frac{x}{2}+\sin\frac{x}{2}\right)dx\approx 2{,}003$

4. $y'=x-y+\cos 2y \qquad x=-4, \qquad y=0$

$y'=x-y+\cos 2y=-3$

$y''=1-y'-2\sin 2y\cdot y'=4$

$y'''=-y''-4\cos 2y\cdot(y')^2-2\sin 2y\cdot y''=-40$

$y^{IV}=-y'''+8\sin 2y\cdot(y')^3-12\cos 2y\cdot y'\cdot y''-2\sin 2y\cdot y'''=184$

Ответ: $y=-\frac{3}{1!}(x+4)+\frac{4}{2!}(x+4)^2-\frac{40}{3!}(x+4)^3+\frac{184}{4!}(x+4)^4+\ldots$

5. $(x^2+1)y''-2xy'+2y=\ln(1+x^2)+\frac{1-3x^2}{1+x^2}$

$x=0; \quad y=0; \quad y'=1 \qquad y|_{x=2/3}=?$

$$y=x+\frac{1}{2}\left[x^2-\frac{x^4}{2}+\frac{x^6}{3}-\frac{x^8}{4}+\ldots+(-1)^{n+1}\cdot\frac{x^{2n}}{n}+\ldots\right]$$

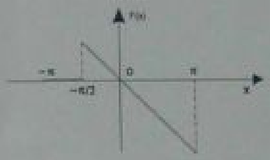
$$y|_{x=2/3}=\frac{2}{3}+\frac{1}{2}\left[\frac{2^2}{3^2}-\frac{2^4}{3^4\cdot 2}+\frac{2^6}{3^6\cdot 3}-\frac{2^8}{3^8\cdot 4}+\ldots+(-1)^{n+1}\cdot\frac{2^{2n}}{3^{2n}\cdot n}+\ldots\right]\approx$$

$$\approx\frac{2}{3}+\frac{1}{2}\left(\frac{4}{9}-\frac{16}{162}+\frac{61}{2187}\right)\approx 0{,}85$$

$$|R|<\frac{1}{2}\cdot\frac{2^8}{3^8\cdot 4}=\frac{2^5}{3^8}=\frac{32}{6561}<0{,}005$$

Ответ: $y=x+\frac{1}{2}\left[x^2-\frac{x^4}{2}+\frac{x^6}{3}-\frac{x^8}{4}+\ldots+(-1)^{n+1}\cdot\frac{x^{2n}}{n}+\ldots\right]$

$y|_{x=2/3}\approx 0{,}85$



6. $f(x)=\begin{cases} 0 & -\pi<x<-\dfrac{\pi}{2} \\ -x & -\dfrac{\pi}{2}<x<\pi \end{cases}$

$$a_0=\frac{1}{\pi}\cdot\left(\int_{-\pi}^{-\pi/2}0\cdot dx+\int_{-\pi/2}^{\pi}(-x)dx\right)=-\frac{3}{8}\pi$$

$$a_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{-\pi/2}0\cdot\cos nxdx+\int_{-\pi/2}^{\pi}(-x)\cos nxdx\right]=$$

$$=\frac{1}{\pi}\left[-\frac{x}{n}\sin nx-\frac{1}{n^2}\cos nx\right]_{-\pi/2}^{\pi}=\frac{1}{\pi}\left[\frac{\pi}{2n}\sin\frac{n\pi}{2}+\frac{1}{n^2}\cos\frac{n\pi}{2}-\frac{(-1)^n}{n^2}\right]$$

$$b_n=\frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2}0\cdot\sin nxdx+\int_{-\pi/2}^{\pi}(-x)\cdot\sin nxdx\right]=\frac{1}{\pi}\left[\frac{x}{n}\cos nx-\frac{1}{n^2}\sin nx\right]_{-\frac{\pi}{2}}^{\pi}=$$

$$=\frac{1}{\pi}\left[\frac{\pi}{n}\cdot(-1)^n+\frac{\pi}{2n}\cdot\cos\frac{n\pi}{2}-\frac{1}{n^2}\sin\frac{n\pi}{2}\right]$$

<u>Ответ:</u> $$f(x)=-\frac{3}{16}\pi+\frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\left[\frac{\pi}{2n}\sin\frac{n\pi}{2}+\frac{1}{n^2}\cos\frac{n\pi}{2}-\frac{(-1)^n}{n^2}\right]\cos nx+$$
$$+\left[\frac{\pi}{n}\cdot(-1)^n+\frac{\pi}{2n}\cos\frac{n\pi}{2}-\frac{1}{n^2}\sin\frac{n\pi}{2}\right]\sin nx$$

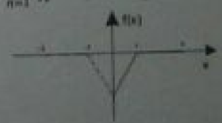
7. $f(x)=\begin{cases} 3x-3 & 0<x<1 \\ 0 & 1<x<3 \end{cases}$ по косинусам

$$a_0=\frac{2}{3}\cdot\int_0^1(3x-3)dx=-1$$

$$a_n=\frac{2}{3}\cdot\int_0^1(3x-3)\cos\frac{n\pi x}{3}dx=2\cdot\left[\frac{3x}{n\pi}\sin\frac{n\pi x}{3}+\frac{9}{n^2\pi^2}\cos\frac{n\pi x}{3}-\right.$$

$$\left.-\frac{3}{n\pi}\sin\frac{n\pi x}{3}\right]_0^1=\frac{18}{n^2\pi^2}\left(\cos\frac{n\pi}{3}-1\right)$$

<u>Ответ:</u> $$f(x)=-\frac{1}{2}+\frac{18}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left(\cos\frac{n\pi}{3}-1\right)\cdot\cos\frac{\pi nx}{3}$$



Вариант 17.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(x+5)^n}{n+2^n}$

$\lim_{n\to\infty}\frac{|x+5|^{n+1}\cdot(n+2^n)}{[(n+1)+2^{n+1}]\cdot|x+5|^n}=\frac{1}{2}\cdot|x+5|<1 \qquad |x+5|<2$

$x+5=2 \qquad \sum_{n=1}^{\infty}(-1)^n\cdot\frac{2^n}{n+2^n}$ - расходится $(\lim_{n\to\infty}U_n\neq 0)$

$x+5=-2 \qquad \sum_{n=1}^{\infty}\frac{2^n}{n+2^n}$ - расходится

Ответ: $-7<x<-3$

2. $f(x)=x\cos 3x$ по степеням $x-\frac{\pi}{3}$

$$x\cos 3x=\left[\left(x-\frac{\pi}{3}\right)+\frac{\pi}{3}\right]\cdot\cos\left[3\cdot\left(x-\frac{\pi}{3}\right)+\pi\right]=-\left[\left(x-\frac{\pi}{3}\right)+\frac{\pi}{3}\right]\cdot\cos 3\cdot\left(x-\frac{\pi}{3}\right)=$$

$$=\frac{\pi}{3}\cdot\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{3^{2n}\cdot\left(x-\frac{\pi}{3}\right)^{2n}}{(2n)!}+\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{3^{2n}\cdot\left(x-\frac{\pi}{3}\right)^{2n+1}}{(2n)!}=$$

$$=\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{3^{2n}}{(2n)!}\cdot\left[\frac{\pi}{3}\cdot\left(x-\frac{\pi}{3}\right)^{2n}+\left(x-\frac{\pi}{3}\right)^{2n+1}\right]$$

$-\infty<x<+\infty$

Ответ: $f(x)=\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{3^{2n}}{(2n)!}\cdot\left[\frac{\pi}{3}\cdot\left(x-\frac{\pi}{3}\right)^{2n}+\left(x-\frac{\pi}{3}\right)^{2n+1}\right]$

$-\infty<x<+\infty$

3. $\int_{-1}^{0}x^4\ln\left(1+\frac{x^4}{4}\right)dx$

$\ln(1+x)=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{n+1}}{(n+1)}; \qquad x^4\ln\left(1+\frac{x^4}{4}\right)=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{4n+8}}{4^{n+1}\cdot(n+1)}$

$$\int_{-1}^{0}x^4\ln\left(1+\frac{x^4}{4}\right)dx=\sum_{n=0}^{\infty}\frac{x^{4n+9}\cdot(-1)^n}{(4n+9)\cdot 4^{n+1}\cdot(n+1)}\Bigg|_{-1}^{0}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+9)\cdot 4^{n+1}\cdot(n+1)}=$$

$$=\frac{1}{9\cdot 4\cdot 1}-\frac{1}{3\cdot 4^2\cdot 2}+\frac{1}{17\cdot 4^3\cdot 3}-\ldots\approx\frac{1}{9\cdot 4\cdot 1}-\frac{1}{3\cdot 16\cdot 2}\approx 0{,}026$$

$|\delta|<\frac{1}{17\cdot 4^3\cdot 3}=\frac{1}{3264}$

Ответ: $\int_{-1}^{0}x^4\ln\left(1+\frac{x^4}{4}\right)dx\approx 0{,}026$

4. $y'=x^4-y^4+2 \qquad x=-1, \quad y=-1$

$y'=x^4-y^4+2=2$

$y''=4x^3-4y^3\cdot y'=4$

$$y''' = 12x^2 - 12y^2 \cdot (y')^2 - 4y^3 y'' = -20$$

$$y^{IV} = 24x - 24y \cdot (y')^3 - 36y^2 \cdot y' \cdot y'' - 4y^3 \cdot y''' = -200$$

<u>**Ответ:**</u> $y = -1 + \frac{2}{1!}(x+1) + \frac{4}{2!}(x+1)^2 - \frac{20}{3!}(x+1)^3 - \frac{200}{4!}(x+1)^4 + \ldots$

5. $(1+x)y'' + xy' - y = 6x + 6x^2 + 2x^3$

$x = 0; \quad y = 1; \quad y' = -1 \qquad y|_{x=-1} = ?$

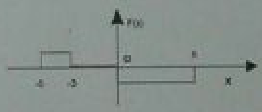
$$\boxed{y = \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \ldots + (-1)^n \cdot \frac{x^n}{n!} + \ldots\right) + x^3}$$

$$y|_{x=-1} = -1 + \left(1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots\right) \approx -1 + \left(1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}\right) \approx 1{,}72$$

$$|\delta| = \frac{1}{n!} + \frac{1}{(n+1)!} + \ldots < \frac{1}{n!}\left(1 + \frac{1}{n} + \frac{1}{n^2} + \ldots\right) = \frac{1}{n!} \cdot \frac{n}{n-1} = \frac{1}{(n-1)!(n-1)}$$

$n = 6 \qquad \delta < \frac{1}{720}$

<u>**Ответ:**</u> $y|_{x=-1} = 1{,}72$



6. $f(x) = \begin{cases} 1 & -5 < x < -3 \\ 0 & -3 < x < 0 \\ -1 & 0 < x < 5 \end{cases}$

$$a_0 = \frac{1}{5} \cdot \left[\int_{-5}^{-3} dx + \int_{-3}^{0} 0 \cdot dx + \int_{0}^{5} (-1) dx\right] = -\frac{3}{5}$$

$$a_n = \frac{1}{5} \cdot \left[\int_{-5}^{-3} \cos\frac{n\pi x}{5} dx + \int_{0}^{5} (-1) \cos\frac{n\pi x}{5} dx\right] = \frac{1}{5} \cdot \left[\frac{5}{n\pi} \sin\frac{n\pi x}{5}\Big|_{-5}^{-3} - \right.$$

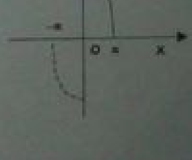
$$\left. - \frac{5}{n\pi} \sin\frac{n\pi x}{5}\Big|_{0}^{5}\right] = -\frac{1}{n\pi} \sin\frac{3n\pi}{5}$$

$$b_n = \frac{1}{5} \cdot \left[\int_{-5}^{-3} \sin\frac{n\pi x}{5} dx + \int_{0}^{5} (-1) \sin\frac{n\pi x}{5} dx\right] = \frac{1}{5} \cdot \left[-\frac{5}{n\pi} \cos\frac{n\pi x}{5}\Big|_{-5}^{-3} + \right.$$

$$\left. + \frac{5}{n\pi} \cos\frac{n\pi x}{5}\Big|_{0}^{5}\right] = \frac{1}{n\pi}\left[2 \cdot (-1)^n - 1 - \cos\frac{3n\pi}{5}\right]$$

<u>**Ответ:**</u>

$$f(x) = -\frac{3}{10} + \frac{1}{\pi} \sum_{n=1}^{\infty} -\frac{1}{n} \sin\frac{3n\pi}{5} \cos\frac{\pi n x}{5} + \frac{1}{n}\left[2 \cdot (-1)^n - 1 - \cos\frac{3n\pi}{5}\right] \sin\frac{n\pi x}{5}$$



7. $f(x) = \pi^2 - x^2 \qquad 0 < x < \pi$ по синусам

$$b_n = \frac{2}{\pi}\cdot\int_0^{\pi}(\pi^2 - x^2)\sin nx dx = \frac{2}{\pi}\left[-\frac{\pi^2}{n}\cos nx + \frac{x^2}{n}\cos nx - \frac{2x}{n^2}\sin nx - \right.$$

$$\left. - \frac{2}{n^3}\cos nx\right]_0^{\pi} = \frac{2}{n\pi}\left[\pi^2 + \frac{2}{n^2} - \frac{2}{n^2}\cdot(-1)^n\right]$$

Ответ: $f(x) = \frac{2}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\left[\pi^2 + \frac{2}{n^2} - \frac{2}{n^2}\cdot(-1)^n\right]\cdot\sin nx$

Вариант 18.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(x-4)^n}{n+3^n}$

$$\lim_{n\to\infty}\frac{|x-4|^{n+1}\cdot(n+3^n)}{[(n+1)+3^{n+1}]|x-4|^n} = |x-4| < 3$$

$x - 4 = 3$ $\quad\sum_{n=1}^{\infty}(-1)^n\cdot\frac{3^n}{n+3^n}$ - расходится $\left(\lim_{n\to\infty}U_n \neq 0\right)$

$x - 4 = -3$ $\quad\sum_{n=1}^{\infty}\frac{3^n}{n+3^n}$ - расходится

Ответ: $1 < x < 7$

2. $f(x) = \sin\left(2x + \frac{\pi}{3}\right)$ по степеням x

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{2^{2n+1}\cdot x^{2n+1}}{(2n+1)!} + \frac{2^{2n}\cdot x^{2n}\cdot\sqrt{3}}{(2n)!}\right]\cdot(-1)^n$$

$-\infty < x < +\infty$

Ответ: $f(x) = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{2^{2n+1}\cdot x^{2n+1}}{(2n+1)!} + \frac{2^{2n}\cdot x^{2n}\cdot\sqrt{3}}{(2n)!}\right]\cdot(-1)^n$

$-\infty < x < +\infty$

3. $\int_0^{0,75}\sqrt[3]{1+x^4}dx$; $\quad\sqrt[3]{1+x} = 1 + \frac{1}{3\cdot 1!}x - \frac{2}{3^2\cdot 2!}x^2 + \frac{2\cdot 5}{3^3\cdot 3!}x^3 - \ldots$

$$\sqrt[3]{1+x^4} = 1 + \frac{1}{3\cdot 1!}x^4 - \frac{2}{3^2\cdot 2!}x^8 + \frac{2\cdot 5}{3^3\cdot 3!}x^{12} - \frac{2\cdot 5\cdot 8}{3^4\cdot 4!}x^{16} + \ldots$$

$$\int_0^{0,75}\sqrt[3]{1+x^4}dx = x + \frac{1}{3\cdot 1!}\cdot\frac{x^5}{5} - \frac{2}{3^2\cdot 2!}\cdot\frac{x^9}{9} + \frac{2\cdot 5}{3^3\cdot 3!}\cdot\frac{x^{13}}{13} - \frac{2\cdot 5\cdot 8}{3^4\cdot 4!}\cdot\frac{x^{17}}{17} + \ldots\Bigg|_0^{0,75} =$$

$$= \frac{3}{4} + \frac{1}{3\cdot 1!}\cdot\frac{3^5}{4^5\cdot 5} - \frac{2}{3^2\cdot 2!}\cdot\frac{3^9}{4^9\cdot 9} + \ldots \approx \frac{3}{4} + \frac{1}{3}\cdot\frac{3^5}{4^5\cdot 5} \approx 0{,}766$$

$$|\delta| < \frac{2}{3^2\cdot 2!}\cdot\frac{3^9}{4^9\cdot 9} = \frac{3^5}{4^9} \approx 0{,}0009$$

Ответ: $\int_0^{0,75}\sqrt[3]{1+x^4}dx \approx 0{,}766$

4. $y' = x^3 + \sin y$ $\quad x = 1, \quad y = \frac{\pi}{2}$

$y' = 2$

$y'' = 3x^2 + \cos y \cdot y' = 3$

$y''' = 6x - \sin y \cdot (y')^2 + \cos y \cdot y'' = 2$

$y^{IV} = 6 - \cos y \cdot (y')^3 - \sin y \cdot 3y' \cdot y'' + \cos y \cdot y''' = -12$

<u>**Ответ:**</u> $y = \frac{\pi}{2} + \frac{2}{1!}(x-1) + \frac{3}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 - \frac{12}{4!}(x-1)^4 + \ldots$

5. $(x^2-5)y'' - 2xy' + 2y = (x^2-3)chx - 2xshx$

$x = 0; \quad y = 1; \quad y' = 0 \qquad y|_{x=1} = ?$

$$y = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$$

$$y|_{x=1} = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots + \frac{1}{(2n)!} + \ldots \approx 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} \approx 1{,}54$$

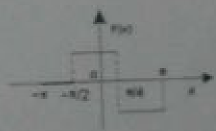
$$|\delta| = \frac{1}{(2n)!} + \frac{1}{(2n+2)!} + \ldots < \frac{1}{(2n)!} \cdot \left[1 + \frac{1}{(2n)^2} + \frac{1}{(2n)^4} + \ldots\right] = \frac{1}{(2n)!} \cdot \frac{(2n)^2}{4n^2 - 1}$$

$n = 3 \qquad \delta < \frac{1}{140}$

<u>**Ответ:**</u> $y = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

$y|_{x=1} = 1{,}54$

6. $$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 1 & -\frac{\pi}{2} < x < \frac{\pi}{4} \\ -1 & \frac{\pi}{4} < x < \pi \end{cases}$$



$$a_0 = \frac{1}{\pi} \cdot \left[\int_{-\pi/2}^{\pi/4} dx + \int_{\pi/4}^{\pi} (-1)dx\right] = 0$$

$$a_n = \frac{1}{\pi} \cdot \left[\int_{-\pi/2}^{\pi/4} \cos nx dx + \int_{\pi/4}^{\pi} (-1)\cos nx dx\right] = \frac{1}{\pi} \cdot \left[\frac{1}{n}\sin nx \Big|_{-\pi/2}^{\pi/4} - \frac{1}{n}\sin nx \Big|_{\pi/4}^{\pi}\right] =$$

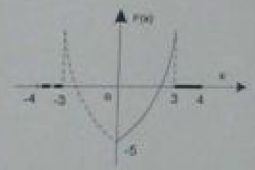
$$= \frac{1}{n\pi}\left[2\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{1}{\pi} \cdot \left[\int_{-\pi/2}^{\pi/4} \sin nx dx + \int_{\pi/4}^{\pi} (-1)\sin nx dx\right] = \frac{1}{\pi} \cdot \left[-\frac{1}{n}\cos nx \Big|_{-\pi/2}^{\pi/4} + \frac{1}{n}\cos nx \Big|_{\pi/4}^{\pi}\right] =$$

$$= \frac{1}{n\pi}\left[(-1)^n + \cos\frac{n\pi}{2} - 2\cos\frac{n\pi}{4}\right]$$

<u>**Ответ:**</u>

$$f(x) = \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \cdot \left[2\sin\frac{n\pi}{4} + \sin\frac{n\pi}{2}\right]\cos nx + \frac{1}{n}\left[(-1)^n + \cos\frac{n\pi}{2} - 2\cos\frac{n\pi}{4}\right]\sin nx$$



7. $f(x)=\begin{cases} x^2-5 & 0<x<3 \\ 0 & 3<x<4 \end{cases}$ по косинусам

$$a_0=\frac{2}{4}\cdot\int_0^3 (x^2-5)dx=-3$$

$$a_n=\frac{2}{4}\cdot\int_0^3 (x^2-5)\cos\frac{n\pi x}{4}dx=\frac{1}{2}\cdot\left[\frac{4x^2}{n\pi}\sin\frac{n\pi x}{4}+\frac{32x}{n^2\pi^2}\cos\frac{n\pi x}{4}-\frac{128}{n^3\pi^3}\sin\frac{n\pi x}{4}-\right.$$

$$\left.-\frac{20}{n\pi}\sin\frac{n\pi x}{4}\right]_0^3=\frac{8}{n\pi}\sin\frac{3n\pi}{4}+\frac{48}{n^2\pi^2}\cos\frac{3n\pi}{4}-\frac{64}{n^3\pi^3}\sin\frac{3n\pi}{4}$$

<u>Ответ:</u> $f(x)=-\frac{3}{2}+8\sum_{n=1}^{\infty}\left[\frac{1}{n\pi}\sin\frac{3n\pi}{4}+\frac{6}{n^2\pi^2}\cos\frac{3n\pi}{4}-\frac{8}{n^3\pi^3}\sin\frac{3n\pi}{4}\right]\cos\frac{n\pi x}{4}$

Вариант 19.

1. $\sum\limits_{n=1}^{\infty}\frac{(x+1)^{2n-1}}{n+\ln(n+1)}$

$\lim\limits_{n\to\infty}\frac{|x+1|^{2n+1}\cdot[n+\ln(n+1)]}{[(n+1)+\ln(n+2)]\cdot|x+1|^{2n-1}}=(x+1)^2;$ $\quad|x+1|<1$

$x+1=1$ $\quad\sum\limits_{n=1}^{\infty}\frac{1}{n+\ln(n+1)}$ $\quad$ расходится (сравнить с $\sum\limits_{n=1}^{\infty}\frac{1}{n}$)

$x+1=-1$ $\quad\sum\limits_{n=1}^{\infty}\frac{-1}{n+\ln(n+1)}$ $\quad$ расходится

<u>**Ответ:**</u> $-2<x<0$

2. $f(x)=ch2x$ $\quad$ по степеням $x-1$

$ch2x=\frac{1}{2}(e^{2x}+e^{-2x})=\frac{e^2}{2}\cdot e^{2(x-1)}+\frac{1}{2e^2}\cdot e^{-2(x-1)}=$

$=\frac{e^2}{2}\sum\limits_{n=0}^{\infty}\frac{2^n(x-1)^n}{n!}+\frac{1}{2e^2}\sum\limits_{n=0}^{\infty}(-1)^n\cdot\frac{2^n(x-1)^n}{n!}=\frac{1}{2}\sum\limits_{n=0}^{\infty}\left[e^2+\frac{(-1)^n}{e^2}\right]\cdot\frac{2^n(x-1)^n}{n!}$

$-\infty<x<+\infty$

<u>**Ответ:**</u> $f(x)=\frac{1}{2}\sum\limits_{n=0}^{\infty}\left[e^2+\frac{(-1)^n}{e^2}\right]\cdot\frac{2^n(x-1)^n}{n!}$

$-\infty<x<+\infty$

3. $\int\limits_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx$ $\qquad\ln(1+x)=\sum\limits_{n=0}^{\infty}(-1)^n\cdot\frac{x^{n+1}}{n+1}$

$\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)=\sum\limits_{n=0}^{\infty}\frac{x^{3n+1}}{10^{n+1}\cdot(n+1)}\cdot(-1)^{2n+1}$

$\int\limits_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx=-\sum\limits_{n=0}^{\infty}\frac{x^{3n+2}}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\Bigg|_0^1=-\sum\limits_{n=0}^{\infty}\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\approx$

$\approx-\left(\frac{1}{2\cdot10\cdot1}+\frac{1}{5\cdot100\cdot2}\right)\approx-0{,}051$

$|\delta|=\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}+\frac{1}{(3n+5)\cdot10^{n+2}\cdot(n+2)}+\ldots<\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)}\times$

$\times\left[1+\frac{1}{10}+\frac{1}{10^2}+\ldots\right]=\frac{1}{(3n+2)\cdot10^{n+1}\cdot(n+1)\cdot9}$

$n=2$ $\qquad\delta<\frac{1}{21600}$

<u>**Ответ:**</u> $\int\limits_0^1\frac{1}{x^2}\ln\left(1-\frac{x^3}{10}\right)dx\approx-0{,}051$

4. $y'=\sqrt{y}-x$ $\qquad x=1$ $\qquad y=4$

$y'=1$

$$y'' = \frac{y'}{2\sqrt{y}} - 1 = -\frac{3}{4}$$

$$y''' = \frac{1}{4}\cdot\frac{2yy'' - y'^2}{y\sqrt{y}} = -\frac{7}{32}$$

$$y^{IV} = \frac{1}{4}\cdot\frac{2y^2y''' - 3yy'y'' + \frac{3}{2}y'^3}{y^2\sqrt{y}} = \frac{7}{256}$$

Ответ: $y(x) = 4 + \frac{1}{1!}\cdot(x-1) - \frac{3}{4\cdot 2!}\cdot(x-1)^2 - \frac{7}{32\cdot 3!}\cdot(x-1)^3 + \frac{7}{256\cdot 4!}(x-1)^4 + \ldots$

5. $(2x^2-1)y'' - 4xy' + 4y = (5-2x^2)\cos x + 4x\sin x$

$x = 0 \qquad y = 1; \qquad y' = 0 \qquad y|_{x=2} = ?$

$$y = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n\cdot\frac{x^{2n}}{(2n)!} + \ldots$$

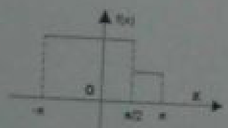
$$y|_{x=2} = 1 - \frac{2^2}{2!} + \frac{2^4}{4!} - \frac{2^6}{6!} + \ldots + (-1)^n\cdot\frac{2^{2n}}{(2n)!} + \ldots \approx 1 - 2 + \frac{16}{24} - \frac{64}{720} \approx -0{,}42$$

$$|\delta| < \frac{256}{40320} < 0{,}01$$

Ответ: $y = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n\cdot\frac{x^{2n}}{(2n)!} + \ldots$

$y|_{x=2} = -0{,}42$

6. $f(x) = \begin{cases} \pi & -\pi < x < \frac{\pi}{2} \\ \frac{\pi}{2} & \frac{\pi}{2} < x < \pi \end{cases}$



$$a_0 = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\pi\cdot dx + \int_{\pi/2}^{\pi}\frac{\pi}{2}dx\right] = \frac{7}{4}\pi$$

$$a_n = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\pi\cos nx dx + \int_{\pi/2}^{\pi}\frac{\pi}{2}\cos nx dx\right] = \frac{1}{\pi}\cdot\left[\frac{\pi}{n}\cdot\sin nx\Big|_{-\pi}^{\pi/2} + \frac{\pi}{2n}\cdot\sin nx\Big|_{\pi/2}^{\pi}\right] =$$

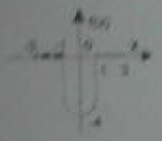
$$= \frac{1}{2n}\cdot\sin\frac{\pi n}{2}$$

$$b_n = \frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\pi\sin nx dx + \int_{\pi/2}^{\pi}\frac{\pi}{2}\sin nx dx\right] = \frac{1}{\pi}\cdot\left[-\frac{\pi}{n}\cos nx\Big|_{-\pi}^{\pi/2} - \frac{\pi}{2n}\cos nx\Big|_{\pi/2}^{\pi}\right] =$$

$$= \frac{1}{2n}\cdot\left[(-1)^n - \cos\frac{n\pi}{2}\right]$$

Ответ: $f(x) = \frac{7}{8}\pi + \frac{1}{2}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{n\pi}{2}\cdot\cos nx + \frac{1}{n}\cdot\left[(-1)^n - \cos\frac{n\pi}{2}\right]\cdot\sin nx$

7. $f(x) = \begin{cases} x^2 - 4 & 0 < x < 1 \\ 0 & 1 < x < 3 \end{cases}$ по косинусам



$$a_0 = \frac{2}{3}\int_0^1 (x^2-4)dx = -\frac{22}{9}$$

$$a_n = \frac{2}{3}\cdot\int_0^1 (x^2-4)\cos\frac{n\pi x}{3}dx =$$

$$= \frac{2}{3}\cdot\left[\frac{3x^2}{\pi n}\sin\frac{n\pi x}{3} + \frac{18x}{\pi^2 n^2}\cos\frac{n\pi x}{3} - \frac{54}{\pi^3 n^3}\sin\frac{n\pi x}{3} - \frac{12}{\pi n}\sin\frac{n\pi x}{3}\right]_0^1 =$$

$$= \frac{2}{n\pi}\cdot\left[\frac{6}{\pi n}\cos\frac{n\pi}{3} - \frac{18}{n^2\pi^2}\sin\frac{n\pi}{3} - 3\sin\frac{n\pi}{3}\right]$$

<u>**Ответ:**</u> $f(x) = -\frac{11}{9} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{6}{\pi n}\cos\frac{n\pi}{3} - \frac{18}{n^2\pi^2}\sin\frac{n\pi}{3} - 3\sin\frac{n\pi}{3}\right]\cdot\cos\frac{n\pi x}{3}$

Вариант 20.

1. $\sum_{n=1}^{\infty}\frac{(x-1)^{3n-1}}{\sqrt{n}+\ln(n+1)}$

$\lim_{n\to\infty}\frac{|x-1|^{3n+2}(\sqrt{n}+\ln(n+1))}{(\sqrt{n+1}+\ln(n+2))|x-1|^{3n-1}} = |x-1|^3$ $\qquad |x-1|<1$

$x-1=1 \qquad \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\ln(n+1)}$ расходится (сравнить с $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$)

$x-1=-1 \qquad \sum_{n=1}^{\infty}\frac{-1}{\sqrt{n}+\ln(n+1)}$ расходится

<u>**Ответ:**</u> $0<x<2$

2. $f(x) = sh3x$ $\qquad$ по степеням $x+1$

$$ch3x = \frac{1}{2}(e^{3x}-e^{-3x}) = \frac{1}{2e^3}\cdot e^{3(x+1)} - \frac{e^3}{2}\cdot e^{-3(x+1)} =$$

$$= \frac{1}{2e^3}\sum_{n=0}^{\infty}\frac{3^n(x+1)^n}{n!} - \frac{e^3}{2}\sum_{n=0}^{\infty}(-1)^n\cdot\frac{3^n(x+1)^n}{n!} = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{e^3}-(-1)^n e^3\right]\cdot\frac{3^n(x+1)^n}{n!}$$

$-\infty<x<+\infty$

<u>**Ответ:**</u> $f(x) = \frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{1}{e^3}-(-1)^n\cdot e^3\right]\cdot\frac{3^n\cdot(x+1)^n}{n!}$

$-\infty<x<+\infty$

3. $\int_0^1 \ln\left(1-\frac{x^5}{5}\right)dx$ $\qquad \ln(1+x) = \sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^{n+1}}{n+1}$

$$\ln\left(1-\frac{x^5}{5}\right) = \sum_{n=0}^{\infty}(-1)^{2n+1}\cdot\frac{x^{5n+5}}{5^{n+1}\cdot(n+1)};$$

$$\int_0^1 \ln\left(1-\frac{x^5}{5}\right)dx = -\sum_{n=0}^{\infty}\frac{x^{5n+6}}{(5n+6)\cdot 5^{n+1}\cdot(n+1)}\Bigg|_0^1 = -\sum_{n=0}^{\infty}\frac{1}{(5n+6)\cdot 5^{n+1}\cdot(n+1)} =$$

$$\approx\left(\frac{1}{6\cdot5\cdot1}+\frac{1}{11\cdot25\cdot2}\right)\approx-0{,}035$$

$$|\delta|=\frac{1}{(5n+6)\cdot5^{n+1}\cdot(n+1)}+\frac{1}{(5n+11)\cdot5^{n+2}\cdot(n+2)}+\ldots<$$

$$<\frac{1}{(5n+6)\cdot5^{n+1}\cdot(n+1)}\cdot\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots\right)=\frac{1}{(5n+6)\cdot5^{n}\cdot(n+1)\cdot4};$$

$n=2$ $\qquad\qquad$ $\delta<\frac{1}{4800}$

Ответ: $\int\limits_0^1 \ln\left(1-\frac{x^5}{5}\right)dx\approx-0{,}035$

4. $y'=(2x-y)^3$ $\qquad\qquad$ $x=2,\quad y=3$

$y'=1$

$y''=3(2x-y)^2\cdot(2-y')=3$

$y'''=6(2x-y)(2-y')^2-3(2x-y)^2\cdot y''=-3$

$y^{IV}=6(2-y')^3-18(2x-y)(2-y')\cdot y''-3(2x-y)^2\cdot y'''=-39$

Ответ: $3+\frac{1}{1!}(x-2)+\frac{3}{2!}(x-2)^2-\frac{3}{3!}(x-2)^3-\frac{39}{4!}(x-2)^4+\ldots=y$

5. $(2x^2-1)y''-4xy'+4y=(5-2x^2)\sin x-4x\cos x$

$x=0,\quad y=0,\quad y'=1,\quad y|_{x=1}=?$

$$y=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\cdot\frac{x^{2n+1}}{(2n-1)!}+\ldots$$

$$y|_{x=1}=1-\frac{1}{3!}+\frac{1}{5!}-\ldots+\frac{(-1)^n}{(2n+1)!}+\ldots\approx1-\frac{1}{6}\approx0{,}83$$

$$|\delta|<\frac{1}{5!}=\frac{1}{120}$$

Ответ: $y=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\cdot\frac{x^{2n+1}}{(2n-1)!}+\ldots$

$y|_{x=1}=0{,}83$

6. $f(x)=\begin{cases}0 & -2<x<0\\ x^2-4 & 0<x<2\end{cases}$

$$a_0=\frac{1}{2}\int\limits_0^2(x^2-4)dx=-\frac{8}{3}$$

$$a_n=\frac{1}{2}\int\limits_0^2(x^2-4)\frac{n\pi x}{2}dx=$$

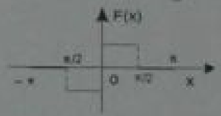
$$=\frac{1}{2}\left(\frac{2x^2}{n\pi}\sin\frac{n\pi x}{2}+\frac{8x}{n^2\pi^2}\cos\frac{n\pi x}{2}-\frac{16}{n^3\pi^3}\sin\frac{n\pi x}{2}-\frac{8}{n\pi}\sin\frac{n\pi x}{2}\right)\Bigg|_0^2=\frac{8}{n^2\pi^2}\cdot(-1)^n$$

$$b_n = \frac{1}{2}\int_0^2 (x^2-4)\sin\frac{n\pi x}{2}\,dx = \frac{1}{2}\left(-\frac{2x^2}{n\pi}\cos\frac{n\pi x}{2} + \frac{8x}{n^2\pi^2}\sin\frac{n\pi x}{2} + \right.$$

$$\left. + \frac{16}{n^3\pi^3}\cos\frac{n\pi x}{2} + \frac{8}{n\pi}\cos\frac{n\pi x}{2}\right)\Bigg|_0^2 = -\frac{4}{n\pi}\left[1 + \frac{2}{n^2\pi^2} - \frac{2}{n^2\pi^2}\cdot(-1)^n\right]$$

Ответ:

$$f(x) = -\frac{4}{3} + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\cdot(-1)^n\cos\frac{n\pi x}{2} - \frac{4}{n\pi}\cdot\left[1 + \frac{2}{n^2\pi^2} - \frac{2}{n^2\pi^2}(-1)^n\right]\cdot\sin\frac{\pi n x}{2}.$$



7. $f(x) = \begin{cases} 1 & 0 < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases}$ по синусам

$$b_n = \frac{2}{\pi}\cdot\int_0^{\pi/2}\sin nx\,dx = -\frac{2}{\pi n}\cos nx\Bigg|_0^{\pi/2} = \frac{2}{\pi n}\cdot\left(1 - \cos\frac{n\pi}{2}\right)$$

Ответ: $f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \cos\frac{n\pi}{2}\right)\cdot\sin nx$

Вариант 21.

$$\sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{n+1}{2n+3}\right)^n\cdot(x+3)^n$$

$\lim\limits_{n\to\infty}\sqrt[n]{\left(\frac{n+1}{2n+3}\right)^n|x+3|^n} = \frac{1}{2}|x+3|^n$; $\quad |x+3| < 2$

$x+3 = 2 \quad \sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{2n+2}{2n+3}\right)^n$ расходится $\quad (\lim\limits_{n\to\infty}U_n = e^{-1/2} \neq 0)$

$x+3 = -2 \quad \sum_{n=1}^{\infty}(-1)^{2n}\cdot\left(\frac{2n+2}{2n+3}\right)^n$ расходится

Ответ: $-5 < x < -1$

2. $f(x) = \sqrt[3]{x}$ по степеням $(x+8)$

$$\sqrt[3]{x} = \sqrt[3]{(x+8)-8} = -2\left(1 - \frac{x+8}{8}\right)^{1/3} =$$

$$= -2\left[1 - \frac{1}{3}\cdot\frac{x+8}{8} - \frac{1}{3}\cdot\frac{3}{2}\cdot\frac{(x+8)^2}{8^2\cdot 2!} - \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{5}{3}\cdot\frac{(x+8)^3}{8^3\cdot 3} - \ldots\right] =$$

$$= -2 + \frac{2}{3}\cdot\frac{x+8}{8} + \frac{2}{3}\cdot\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot\ldots\cdot(3n-1)}{8^{n+1}\cdot 3^n}\cdot\frac{(x+8)^{n+1}}{(n+1)!}$$

$-1 \le \frac{x+8}{8} \le 1 \qquad -16 \le x \le 0$

Ответ: $f(x) = -2 + \frac{2(x+8)}{24} + \frac{2}{3}\cdot\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot\ldots\cdot(3n-1)}{8^{n+1}\cdot 3^n}\cdot\frac{(x+8)^{n+1}}{(n+1)!}$

$-16 \le x \le 0$

3. $\int\limits_0^1 e^{\frac{x^2}{10}}dx$

$e^x=\sum\limits_{n=0}^{\infty}\frac{x^n}{n!}$; $\qquad e^{\frac{x^2}{10}}=\sum\limits_{n=0}^{\infty}\frac{x^{2n}}{10^n\cdot n!}$

$$\int\limits_0^1 e^{\frac{x^2}{10}}dx=\sum\limits_{n=0}^{\infty}\frac{x^{2n+1}}{10^n\cdot n!\cdot(2n+1)}\Bigg|_0^1=\sum\limits_{n=0}^{\infty}\frac{1}{10^n\cdot n!\cdot(2n+1)}\approx 1+\frac{1}{10\cdot1\cdot3}+\frac{1}{100\cdot2\cdot5}\approx 1{,}034$$

$$\delta=\frac{1}{(2n+1)\cdot10^n\cdot n!}\cdot\left[1+\frac{1}{10n}+\frac{1}{(10n)^2}+\ldots\right]=\frac{1}{(2n+1)\cdot10^n\cdot n!}\cdot\frac{10n}{10n-1}=$$
$$=\frac{1}{(2n+1)\cdot10^{n-1}\cdot(n-1)!\cdot(10n-1)}$$

$n=3 \qquad \delta<\frac{1}{40600}$

Ответ: $\int\limits_0^1 e^{\frac{x^2}{10}}dx\approx 1{,}034$

4. $y'=e^{x-y}-x \qquad x=2, \quad y=2$

$y'=-1$

$y''=e^{x-y}\cdot(1-y')-1=1$

$y'''=e^{x-y}\cdot(1-y')^2-e^{x-y}\cdot y''=3$

$y^{IV}=e^{x-y}\cdot(1-y')^3-3e^{x-y}\cdot(1-y')\cdot y''-e^{x-y}\cdot y'''=-1$

Ответ: $y=2-\frac{1}{1!}(x-2)+\frac{1}{2!}(x-2)^2+\frac{3}{3!}(x-2)^3-\frac{1}{4!}(x-2)^4+\ldots$

5. $(x^2+1)y''-2xy'+2y=(1-x^2)\sin x-2x\cos x$

$x=0, \quad y=0, \quad y'=1, \qquad y|_{x=2}=?$

$$y=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots+(-1)^n\cdot\frac{x^{2n+1}}{(2n+1)!}+\ldots$$

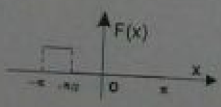
$$y|_{x=2}=2-\frac{2^3}{3!}+\frac{2^5}{5!}-\ldots+(-1)^n\cdot\frac{2^{2n+1}}{(2n+1)!}+\ldots\approx2-\frac{8}{6}+\frac{32}{120}-\frac{128}{5040}\approx0{,}91$$

$|\delta|<\frac{2^9}{5!}<0{,}002$

Ответ: $y=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots+(-1)^n\cdot\frac{x^{2n+1}}{(2n-1)!}+\ldots$

$y|_{x=2}=0{,}91$

6. $f(x)=\begin{cases}1 & -\pi<x<-\frac{\pi}{2}\\ 0 & -\frac{\pi}{2}<x<\pi\end{cases}$

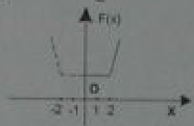


$a_0=\frac{1}{\pi}\cdot\int\limits_{-\pi}^{-\pi/2}dx=\frac{1}{2}$

$$a_n=\frac{1}{\pi}\cdot\int_{-\pi}^{-\pi/2}\cos nx dx=\frac{1}{n\pi}\cdot\sin nx\Big|_{-\pi}^{-\pi/2}=-\frac{1}{n\pi}\cdot\sin\frac{n\pi}{2}$$

$$\int_{-\pi}^{-\pi/2}\sin nx dx=-\frac{1}{n\pi}\cos nx\Big|_{-\pi}^{-\pi/2}=-\frac{1}{n\pi}\left[\cos\frac{n\pi}{2}-(-1)^n\right]$$

$$f(x)=\frac{1}{4}-\frac{1}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{n\pi}{2}\cdot\cos nx+\frac{1}{n}\cdot\left[\cos\frac{n\pi}{2}-(-1)^n\right]\cdot\sin nx$$



$$f(x)=\begin{cases}1 & 0<x<1\\ x^2 & 1<x<2\end{cases}$$ по косинусам

$$\ldots\frac{2}{2}\cdot\left(\int_0^1 dx+\int_1^2 x^2dx\right)=\frac{10}{3}$$

$$=\frac{2}{2}\cdot\left(\int_0^1\cos\frac{n\pi x}{2}dx+\int_1^2 x^2\cos\frac{n\pi x}{2}dx\right)=\frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^1+$$

$$+\left(\frac{2x^2}{n\pi}\sin\frac{n\pi x}{2}+\frac{8x}{n^2\pi^2}\cos\frac{n\pi x}{2}-\frac{16}{n^3\pi^3}\sin\frac{n\pi x}{2}\right)\Big|_1^2=$$

$$=\frac{8}{n^2\pi^2}\cdot\left[2\cdot(-1)^n-\cos\frac{n\pi}{2}+\frac{2}{n\pi}\sin\frac{n\pi}{2}\right]$$

Ответ: $f(x)=\frac{5}{3}+\frac{8}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left[2\cdot(-1)^n-\cos\frac{n\pi}{2}+\frac{2}{n\pi}\sin\frac{\pi n}{2}\right]\cdot\cos\frac{n\pi x}{2}$

Вариант 22.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\left(x-\frac{3}{2}\right)^{2n}}{\sqrt{n}+4^n}$

$$\lim_{n\to\infty}\frac{\left|x-\frac{3}{2}\right|^{2n+2}\cdot(\sqrt{n}+4^n)}{(\sqrt{n+1}+4^{n+1})\cdot\left|x-\frac{3}{2}\right|^{2n}}=\frac{\left(x-\frac{3}{2}\right)^2}{4};\qquad \left|x-\frac{3}{2}\right|<2$$

$x-\frac{3}{2}=\pm2\qquad \sum_{n=1}^{\infty}(-1)^n\cdot\frac{4^n}{\sqrt{n}+4^n}$ расходится $\left(\sum_{n\to\infty}U_n\neq0\right)$

Ответ: $-\frac{1}{2}<x<\frac{7}{2}$

2. $f(x)=\cos\left(2x+\frac{\pi}{4}\right)$ по степеням $\left(x-\frac{\pi}{4}\right)$

$$\cos\left(2x+\frac{\pi}{4}\right)=\cos\left[2\cdot\left(x-\frac{\pi}{4}\right)+\frac{3\pi}{4}\right]=-\frac{\sqrt{2}}{2}\cdot\left[\cos2\cdot\left(x-\frac{\pi}{4}\right)+\sin2\cdot\left(x-\frac{\pi}{4}\right)\right]=$$

$$=-\frac{\sqrt{2}}{2}\cdot\left[\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n}\cdot\left(x-\frac{\pi}{4}\right)^{2n}}{(2n)!}+\sum_{n=0}^{\infty}(-1)^n\cdot\frac{2^{2n+1}\cdot\left(x-\frac{\pi}{4}\right)^{2n+1}}{(2n+1)!}\right]=$$

$$=-\frac{\sqrt{2}}{2}\cdot\sum_{n=0}^{\infty}\frac{2^n\cdot\left(x-\frac{\pi}{4}\right)^n}{n!}\cdot(-1)^{\frac{n^2-n}{2}}\qquad -\infty<x<+\infty$$

<u>**Ответ:**</u> $f(x)=-\frac{\sqrt{2}}{2}\cdot\sum_{n=0}^{\infty}\frac{2^n\cdot\left(x-\frac{\pi}{4}\right)^n}{n!}\cdot(-1)^{\frac{n^2-n}{2}}$

$-\infty<x<+\infty$

3. $\int_0^{1/2}\sqrt{1+x^3}\,dx$

$$\sqrt{1+x}=1+\frac{1}{2\cdot 1!}x-\frac{1\cdot 1}{2^2\cdot 2!}x^2+\frac{1\cdot 1\cdot 3x^3}{2^3\cdot 3!}-\frac{1\cdot 1\cdot 3\cdot 5}{2^4\cdot 4!}x^4+\ldots$$

$$\sqrt{1+x^3}=1+\frac{1}{2\cdot 1!}x^3-\frac{1\cdot 1}{2^2\cdot 2!}x^6+\frac{1\cdot 3}{2^3\cdot 3!}x^9-\frac{1\cdot 3\cdot 5}{2^4\cdot 4!}x^{12}+\ldots$$

$$\int_0^{1/2}\sqrt{1+x^3}\,dx=x+\frac{1}{2\cdot 1!}\cdot\frac{x^4}{4}-\frac{1}{2^2\cdot 2!}\cdot\frac{x^7}{7}+\frac{1\cdot 3}{2^3\cdot 3!}\cdot\frac{x^{10}}{10}-\frac{1\cdot 3\cdot 5}{2^4\cdot 4!}\cdot\frac{x^{13}}{13}+\ldots\Big|_0^{1/2}=$$

$$=\frac{1}{2}+\frac{1}{2\cdot 1!}\cdot\frac{1}{4\cdot 2^4}-\frac{1}{2^2\cdot 2!}\cdot\frac{1}{7\cdot 2^7}+\ldots\approx\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2^4\cdot 4}\approx 0{,}508$$

$$|\delta|<\frac{1}{2^2\cdot 2\cdot 7\cdot 2^7}=\frac{1}{1024\cdot 7}$$

<u>**Ответ:**</u> $\int_0^{1/2}\sqrt{1+x^3}\,dx=0{,}508$

4. $y'=x^2+2\sqrt{y}\qquad x=2,\qquad y=1$

$y'=6$

$$y''=2x+\frac{y'}{\sqrt{y}}=10$$

$$y'''=2+\frac{2yy''-y'^2}{2y\sqrt{y}}=-6$$

$$y^{IV}=\frac{1}{2}\cdot\frac{2y^2y'''-3yy'y''+\frac{3}{2}y'^3}{y^2\sqrt{y}}=82$$

<u>**Ответ:**</u> $y=1+\frac{6}{1!}\cdot(x-2)+\frac{10}{2!}\cdot(x-2)^2-\frac{6}{3!}\cdot(x-2)^3+\frac{82}{4!}(x-2)^4+\ldots$

5. $(x^2+1)y''-2xy'+2y=e^x\cdot(x^2-2x+3)$

$x=0,\qquad y=1,\qquad y'=1\qquad y|_{x=1}=?$

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$y|_{x=1} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} \approx 2{,}72$$

$$\delta = \frac{1}{n!} + \frac{1}{(n+1)!} + \ldots < \frac{1}{n!} \cdot \left(1 + \frac{1}{n} + \frac{1}{n^2} + \ldots\right) = \frac{1}{n!} \cdot \frac{n}{n-1} = \frac{1}{(n-1)! \cdot (n-1)}$$

$n = 6$ $\qquad \delta < \frac{1}{720}$

Ответ: $\quad y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \frac{x^n}{n!} + \ldots$

$\quad y|_{x=1} = 2{,}72$

6. $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi} \cdot \int_0^{\pi} x^2 dx = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \cdot \int_0^{\pi} x^2 \cos nx dx = \frac{1}{\pi}\left[\frac{x^2}{n} \sin nx + \frac{2x}{n^2} \cos nx - \frac{2}{n^3} \sin nx\right]_0^{\pi} = \frac{2}{n^2} \cdot (-1)^n$$

$$b_n = \frac{1}{\pi} \cdot \int_0^{\pi} x^2 \sin nx dx = \frac{1}{\pi}\left[-\frac{x^2}{n} \cos nx + \frac{2x}{n^2} \sin nx + \frac{2}{n^3} \cos nx\right]_0^{\pi} =$$

$$= \frac{1}{\pi} \cdot \left[\frac{2}{n^3} \cdot (-1)^n - \frac{2}{n^3} - \frac{\pi^2}{n} \cdot (-1)^n\right]$$

Ответ:

$$f(x) = \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{2}{n^2} \cdot (-1)^n \cdot \cos nx + \frac{1}{\pi} \cdot \left[\frac{2}{n^3} \cdot (-1)^n - \frac{2}{n^3} - \frac{\pi^2}{n} \cdot (-1)^n\right] \cdot \sin nx$$

7. $f(x) = \begin{cases} 0 & 0 < x < 2 \\ -1 & 2 < x < 5 \end{cases}$ по косинусам

$$a_0 = \frac{2}{5} \cdot \int_2^5 -dx = -\frac{6}{5}$$

$$a_n = \frac{2}{5} \cdot \int_2^5 -\cos\frac{n\pi x}{5} dx = -\frac{2}{\pi n} \sin\frac{n\pi x}{5}\Big|_2^5 = \frac{2}{\pi n} \sin\frac{2n\pi}{5}$$

Ответ: $\quad f(x) = -\frac{3}{5} + \frac{2}{\pi} \cdot \sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{2n\pi}{5} \cdot \cos\frac{n\pi x}{5}$

Вариант 23.

1. $\sum\limits_{n=1}^{\infty} n\sin\frac{1}{n}(x-5)^{2n-1}$

$$\lim_{n\to\infty}\frac{(n+1)\sin\frac{1}{n+1}|x-5|^{2n+1}}{n\sin\frac{1}{n}|x-5|^{2n-1}}=(x-5)^2; \qquad |x-5|<1$$

$|x-5|=1 \qquad \sum\limits_{n=1}^{\infty} n\sin\frac{1}{n}$ расходится $\quad (\lim\limits_{n\to\infty} U_n \neq 0)$

$x-5=-1 \qquad -\sum\limits_{n=1}^{\infty} n\sin\frac{1}{n}$ расходится

__Ответ:__ $4<x<6$

2. $f(x)=\frac{1}{x(x-1)}$ по степеням $(x+2)$

$$\frac{1}{x(x-1)}=\frac{1}{x-1}-\frac{1}{x}=-\frac{1}{3-(x+2)}+\frac{1}{2-(x+2)}=-\frac{1}{3}\cdot\frac{1}{1-\frac{x+2}{3}}+\frac{1}{2}\cdot\frac{1}{1-\frac{x+2}{2}}=$$

$$=-\sum_{n=0}^{\infty}\frac{(x+2)^2}{3^{n+1}}+\sum_{n=0}^{\infty}\frac{(x+2)^n}{2^{n+1}}=\sum_{n=0}^{\infty}\left(\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}\right)\cdot(x+2)^2$$

$-2<x+2<2; \qquad -4<x<0$

__Ответ:__ $f(x)=\sum\limits_{n=0}^{\infty}\left(\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}\right)\cdot(x+2)^n$

$-4<x<0$

3. $\int\limits_0^1 x^3 ch\frac{x}{3}dx$

$$chx=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}; \quad x^3 ch\frac{x}{3}=\sum_{n=0}^{\infty}\frac{x^{2n+3}}{3^{2n}\cdot(2n)!}$$

$$\int_0^1 x^3 ch\frac{x}{3}dx=\sum_{n=0}^{\infty}\frac{x^{2n+6}}{3^{2n}\cdot(2n)!(2n+6)}\Bigg|_0^1=\sum_{n=0}^{\infty}\frac{1}{3^{2n}\cdot(2n)!(2n+6)}\approx\frac{1}{6}+\frac{1}{144}=0{,}174$$

$$\delta=\frac{1}{3^{2n}\cdot(2n)!(2n+6)}+\frac{1}{3^{2n+2}\cdot(2n+2)!(2n+8)}+\ldots<\frac{1}{3^{2n}\cdot(2n)!(2n+6)}\times$$

$$\times\left[1+\frac{1}{9\cdot(2n)^2}+\frac{1}{9^2\cdot(2n)^4}+\ldots\right]=\frac{1}{3^{2n}\cdot(2n)!(2n+6)}\cdot\frac{9\cdot(2n)^2}{(36n^2-1)}$$

$n=2 \qquad \delta<\frac{1}{135\cdot143}=\frac{1}{19305}$

__Ответ:__ $\int\limits_0^1 x^3 ch\frac{x}{3}dx=0{,}174$

4. $y'=y\sqrt{y}-4x \qquad x=1, \quad y=4$

$y'=4$

$$y'' = \frac{3}{2} y' \sqrt{y} - 4 = 8$$

$$y''' = \frac{3}{2}\left(\frac{y'^2}{2\sqrt{y}} + \sqrt{y}\,y''\right) = 30$$

$$y^{IV} = \frac{3}{2}\left(\frac{4yy'y'' - y'^3}{4y\sqrt{y}} + \frac{y'y''}{2\sqrt{y}} + \sqrt{y}\,y'''\right) = 123$$

Ответ: $y = 4 + \frac{4}{1!}\cdot(x-1) + \frac{8}{2!}\cdot(x-1)^2 + \frac{30}{3!}\cdot(x-1)^3 + \frac{123}{4!}\cdot(x-1)^4 + \ldots$

5. $(x^2-1)y'' - 2xy' + 2y = e^{-x^2}\cdot(4x^4 - 2x^2 + 4)$

$x = 0, \quad y = 1, \quad y' = 0 \quad y|_{x=1} = ?$

$$y = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots + (-1)^n \cdot \frac{x^{2n}}{n!} + \ldots$$

$$y|_{x=1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \ldots + \frac{(-1)^n}{n!} + \ldots \approx 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} \approx 0{,}37$$

$$|\delta| < \frac{1}{120}$$

Ответ: $y = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots + (-1)^n \cdot \frac{x^{2n}}{n!} + \ldots$

$y|_{x=1} = 0{,}37$

6. $f(x) = \begin{cases} 1 & -5 < x < 0 \\ -\frac{x}{2} & 0 < x < 5 \end{cases}$

$$a_0 = \frac{1}{5}\cdot\left(\int_{-5}^{0} dx - \int_{0}^{5} \frac{x}{2} dx\right) = -\frac{1}{4}$$

$$a_n = \frac{1}{5}\cdot\left(\int_{-5}^{0} \cos\frac{n\pi x}{5} dx - \int_{0}^{5} \frac{x}{2}\cos\frac{n\pi x}{5} dx\right) = \frac{1}{n\pi}\sin\frac{\pi n x}{5}\Big|_{-5}^{0} -$$

$$-\frac{1}{10}\cdot\left(\frac{5x}{n\pi}\sin\frac{n\pi x}{5} + \frac{25}{n^2\pi^2}\cos\frac{n\pi x}{5}\right)\Big|_{0}^{5} = -\frac{5}{2n^2\pi^2}\cdot\left[(-1)^n - 1\right]$$

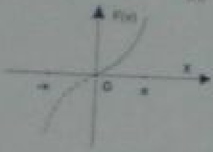
$$b_n = \frac{1}{5}\cdot\left(\int_{-5}^{0} \sin\frac{n\pi x}{5} dx - \int_{0}^{5} \frac{x}{2}\sin\frac{n\pi x}{5} dx\right) = -\frac{1}{n\pi}\cos\frac{\pi n x}{5}\Big|_{-5}^{0} -$$

$$-\frac{1}{10}\cdot\left(-\frac{5x}{n\pi}\cos\frac{n\pi x}{5} + \frac{25}{n^2\pi^2}\sin\frac{n\pi x}{5}\right)\Big|_{0}^{5} = \frac{7\cdot(-1)^n - 2}{2n\pi}$$

Ответ: $f(x) = -\frac{1}{8} + \frac{1}{2\pi}\cdot\sum_{n=1}^{\infty}\frac{5}{n^2\pi}\cdot\left[1 - (-1)^n\right]\cos\frac{n\pi x}{5} + \frac{7\cdot(-1)^n - 2}{n}\sin\frac{n\pi x}{5}$

7. $f(x) = x^2 \qquad 0 < x < \pi$ по синусам

$$b_n=\frac{2}{\pi}\cdot\int_0^{\pi}x^2\sin nx\,dx=\frac{2}{\pi}\cdot\left[-\frac{x^2}{n}\cos nx+\frac{2x}{n^2}\sin nx+\frac{2}{n^3}\cos nx\right]_0^{\pi}=$$



$$=\frac{2}{\pi}\cdot\left[\frac{2}{n^3}(-1)^n-\frac{2}{n^3}-\frac{\pi^2}{n}\cdot(-1)^n\right]$$

<u>**Ответ:**</u> $f(x)=\frac{2}{\pi}\cdot\sum\limits_{n=1}^{\infty}\left[\frac{2}{n^3}(-1)^n-\frac{2}{n^3}-\frac{\pi^2}{n}\cdot(-1)^n\right]\sin nx$

Вариант 24.

1. $\sum\limits_{n=1}^{\infty}\frac{(x+6)^n}{n^2+5^n}$

$\lim\limits_{n\to\infty}\frac{|x+6|^{n+1}(n^2+5^n)}{[(n+1)^2+5^{n+1}]|x+6|^n}=\frac{1}{5}|x+6|;\qquad |x+6|<5$

$x+6=5\qquad \sum\limits_{n=1}^{\infty}\frac{5^n}{n^2+5^n}$ - расходится $(\lim\limits_{n\to\infty}U_n\neq 0)$

$x+6=-5\qquad \sum\limits_{n=1}^{\infty}\frac{(-1)^n\cdot 5^n}{n^2+5^n}$ - расходится $(\lim\limits_{n\to\infty}U_n\neq 0)$

<u>**Ответ:**</u> $-11<x<-1$

2. $f(x)=x\sin 2x$ по степеням $\left(x+\frac{\pi}{4}\right)$

$$x\sin 2x=\left[\left(x+\frac{\pi}{4}\right)-\frac{\pi}{4}\right]\cdot\sin\left[\left(\frac{\pi}{4}+2x\right)-\frac{\pi}{2}\right]=-\left[\left(x+\frac{\pi}{4}\right)-\frac{\pi}{4}\right]\cdot\cos 2\left(x+\frac{\pi}{4}\right)=$$

$$=\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{2^{2n}\cdot\left(x+\frac{\pi}{4}\right)^{2n+1}}{(2n)!}-\frac{\pi}{4}\sum_{n=0}^{\infty}(-1)^{n+1}\cdot\frac{2^{2n}\cdot\left(x+\frac{\pi}{4}\right)^{2n}}{(2n)!}=$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\cdot 2^{2n}}{(2n)!}\cdot\left[\left(x+\frac{\pi}{4}\right)^{2n+1}-\frac{\pi}{4}\cdot\left(x+\frac{\pi}{4}\right)^{2n}\right]$$

$-\infty<x<+\infty$

<u>**Ответ:**</u> $f(x)=\sum\limits_{n=0}^{\infty}\frac{(-1)^{n+1}\cdot 2^{2n}}{(2n)!}\cdot\left[\left(x+\frac{\pi}{4}\right)^{2n+1}-\frac{\pi}{4}\cdot\left(x+\frac{\pi}{4}\right)^{2n}\right]$

$-\infty<x<+\infty$

3. $\int\limits_0^{0,5}\frac{dx}{\sqrt{1+x^3}}$

$$\frac{1}{\sqrt{1+x}}=1-\frac{1}{2\cdot 1!}x+\frac{1\cdot 3}{2^2\cdot 2!}x^2-\frac{1\cdot 3\cdot 5}{2^3\cdot 3!}x^3+\cdots$$

$$\frac{1}{\sqrt{1+x^3}} = 1 - \frac{1}{2\cdot 1!}x^3 + \frac{1\cdot 3}{2^2\cdot 2!}x^6 - \frac{1\cdot 3\cdot 5}{2^3\cdot 3!}x^9 + \cdots$$

$$\int_0^{0,6} \frac{1}{\sqrt{1+x^3}} = x - \frac{1}{2\cdot 1!}\cdot\frac{x^4}{4} + \frac{1\cdot 3}{2^2\cdot 2!}\cdot\frac{x^7}{7} - \frac{1\cdot 3\cdot 5}{2^3\cdot 3!}\cdot\frac{x^{10}}{10} + \ldots\Big|_0^{0,6} \approx$$

$$\approx \frac{3}{5} - \frac{1}{2\cdot 1!}\cdot\frac{3^4}{5^4\cdot 4} + \frac{1\cdot 3}{2^2\cdot 2!}\cdot\frac{3^7}{5^7\cdot 7} \approx 0{,}585$$

$$|\delta| < \frac{1\cdot 3\cdot 5}{2^3\cdot 3!}\cdot\frac{3^{10}}{5^{10}\cdot 10} < 0{,}0002$$

Ответ: $\int_0^{0,6} \frac{dx}{\sqrt{1+x^3}} = 0{,}585$

4. $y' = xy^2 + 6 \qquad x = -1, \qquad y = 2$

$y' = 2$

$y'' = y^2 + 2xyy' = -4$

$y''' = 2yy' + 2yy' + 2xy'^2 + 2xyy'' = 24$

$y^{IV} = 6y'^2 + 6yy'' + 6xy'y'' + 2xyy''' = -72$

Ответ: $y = 2 + \frac{2}{1!}\cdot(x+1) - \frac{4}{2!}\cdot(x+1)^2 + \frac{24}{3!}\cdot(x+1)^3 - \frac{72}{4!}(x+1)^4 + \ldots$

5. $(x^2-1)y'' - 2xy' + 2y = (x^2+2x+1)\cdot e^{-x}$

$x = 0 \qquad y = 1, \qquad y' = -1 \qquad y|_{x=-1} = ?$

$$y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots(-1)^n\cdot\frac{x^n}{n!} + \ldots$$

$$y|_{x=-1} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots \approx 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} \approx 2{,}72$$

$$\delta = \frac{1}{n!} + \frac{1}{(n+1)!} + \ldots < \frac{1}{n!}\left(1 + \frac{1}{n} + \frac{1}{n^2} + \ldots\right) = \frac{1}{(n-1)!\cdot(n-1)}$$

$n = 6 \qquad \delta < \frac{1}{720}$

Ответ: $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots(-1)^n\cdot\frac{x^n}{n!} + \ldots$

$y|_{x=-1} = 2{,}72$

6. $f(x) = \begin{cases} 0 & -4 < x < -2 \\ 3 & -2 < x < 3 \\ 1 & 3 < x < 4 \end{cases}$

$$a_0 = \frac{1}{4}\cdot\left(\int_{-2}^{3} 3dx + \int_3^4 dx\right) = 4$$

$$a_n = \frac{1}{4}\cdot\left(\int_{-2}^{3} 3\cos\frac{n\pi x}{4}dx + \int_3^4 \cos\frac{n\pi x}{4}dx\right) = \frac{1}{4}\cdot\left[\frac{12}{n\pi}\sin\frac{n\pi x}{4}\Big|_{-2}^{3} + \frac{4}{n\pi}\sin\frac{n\pi x}{4}\Big|_3^4\right] =$$

$$= \frac{1}{n\pi}\cdot\left[2\sin\frac{3n\pi}{4}+3\sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{1}{4}\cdot\left(\int_{-2}^{3} 3\sin\frac{n\pi x}{4}dx+\int_{3}^{4}\sin\frac{n\pi x}{4}dx\right)=\frac{1}{4}\cdot\left[-\frac{12}{n\pi}\cos\frac{n\pi x}{4}\Bigg|_{-2}^{3}-\right.$$

$$\left.-\frac{4}{n\pi}\cos\frac{n\pi x}{4}\Bigg|_{3}^{4}\right]=\frac{1}{n\pi}\left(3\cos\frac{n\pi}{2}-\cos\frac{3n\pi}{4}-(-1)^n\right)$$

Ответ: $f(x)=2+\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[2\sin\frac{3n\pi}{4}+3\sin\frac{\pi n}{2}\right]\cos\frac{\pi nx}{4}+$

$$+\frac{1}{n}\left[3\cos\frac{n\pi}{2}-2\cos\frac{3n\pi}{4}-(-1)^n\right]\sin\frac{n\pi x}{4}$$

7. $f(x)=\begin{cases}1-x^2 & 0<x<1\\ 0 & 1<x<\pi\end{cases}$ по косинусам

$$a_0=\frac{2}{\pi}\cdot\int_0^1(1-x^2)dx=\frac{4}{3\pi}$$

$$a_n=\frac{1}{\pi}\cdot\int_0^1(1-x^2)\cos nxdx=\frac{2}{\pi}\cdot\left[\frac{1}{n}\sin nx-\frac{x^2}{n}\sin nx-\frac{2x}{n^2}\cos nx+\frac{2}{n^3}\sin nx\right]_0^1=$$

$$=\frac{4}{\pi n^2}\left(\frac{1}{n}\sin n-\cos n\right)$$

Ответ: $f(x)=\frac{2}{3\pi}+\frac{4}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left(\frac{1}{n}\sin n-\cos n\right)\cdot\cos nx$

Вариант 25.

1. $\sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{n}{9n+2}\right)^n\cdot(x+2)^{2n}$

$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n}{9n+2}\right)^n\cdot|x+2|^{2n}}=\frac{1}{9}|x+2|^2$; $|x+2|<3$

$x+2=\pm3$ $\sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{9n}{9n+2}\right)^n$ расходится $(\lim_{n\to\infty}U_n=e^{-2/9}\neq0)$

Ответ: $-5<x<1$

2. $f(x)=\frac{1}{x^2-2x+3}$ по степеням $x-1$

$$\frac{1}{x^2-2x+3}=\frac{1}{2+(x-1)^2}=\frac{1}{2}\cdot\frac{1}{1+\frac{(x-1)^2}{2}}=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{(x-1)^{2n}}{2^{n+1}}$$

$\frac{(x-1)^2}{2}<1$ $1-\sqrt{2}<x<1+\sqrt{2}$

Ответ: $f(x)=\sum\limits_{n=0}^{\infty}(-1)^n\cdot\frac{(x-1)^{2n}}{2^{n+1}}$

$1-\sqrt{2}<x<1+\sqrt{2}$

3. $\int\limits_0^{1/2}x(\cos 2x+ch2x)dx$

$\cos x=\sum\limits_{n=0}^{\infty}(-1)^n\cdot\frac{x^{2n}}{(2n)!};\qquad chx=\sum\limits_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}.$

[illegible] $2x+ch2x)=2\sum\limits_{n=0}^{\infty}\frac{x^{4n+1}\cdot 2^{4n}}{(4n)!}$

[illegible] $(\cos 2x+ch2x)dx=2\sum\limits_{n=0}^{\infty}\frac{x^{4n+2}\cdot 2^{4n}}{(4n)!\cdot(4n+2)}\Bigg|_0^{1/2}=\frac{1}{4}\sum\limits_{n=0}^{\infty}\frac{1}{(4n)!\cdot(2n+1)}\approx$

[illegible] $\left(1+\frac{1}{4!\cdot 3}\right)\approx 0{,}253$

[illegible] $\frac{1}{4}\left[\frac{1}{(4n)!(2n+1)}+\frac{1}{(4n+4)!(2n+3)}+\ldots\right]<\frac{1}{4}\cdot\frac{1}{(4n)!(2n+1)}\times$

$\times\left[1+\frac{1}{(4n)^4}+\frac{1}{(4n)^{16}}+\ldots\right]=\frac{1}{4}\cdot\frac{1}{(4n)!(2n+1)}\cdot\frac{(4n)^4}{4^4\cdot n^4-1}$

$n=2\qquad \delta<\frac{8}{45\cdot 4090}<\frac{1}{20000}$

Ответ: $\int\limits_0^{1/2}x(\cos 2x+ch2x)dx=0{,}253$

4. $y'=(x-e^y)^2+x\qquad x=2;\quad y=0$

$y'=3$

$y''=2(x-e^y)(1-e^y\cdot y')+1=-3$

$y'''=2(1-e^y\cdot y')^2-2(x-e^y)(e^y y'^2+e^y y'')=-4$

$y^{IV}=-6(1-e^y y')(e^y y'^2+e^y y'')-2(x-e^y)(e^y y'^3+e^y\cdot 3y'y''+e^y y''')=80$

Ответ: $y=\frac{3}{1!}(x-2)-\frac{3}{2!}(x-2)^2-\frac{4}{3!}(x-2)^3+\frac{80}{4!}(x-2)^4+\ldots$

5. $(4x^2-1)y''-8xy$©$+8y=(16x^4+4x^2+10)\cdot e^{-x^2}$

$x=0\qquad y=1,\quad y'=0\qquad y|_{x=1/3}=?$

$y=1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots+(-1)^n\cdot\frac{x^{2n}}{n!}+\ldots$

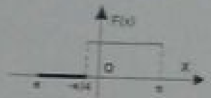
$y|_{x=1/3}=1-\frac{1}{3^2\cdot 1!}+\frac{1}{3^4\cdot 2!}-\frac{1}{3^6\cdot 3!}+\ldots+\frac{(-1)^n}{3^{2n}\cdot n!}+\ldots\approx 1-\frac{1}{9}\approx 0{,}89$

$\delta<\frac{1}{3^4\cdot 2!}=\frac{1}{162}$

Ответ: $y = 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots + (-1)^n \cdot \frac{x^{2n}}{n!} + \ldots$

$y|_{x=1/3} = 0{,}89$

6. $f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{4} \\ \pi & -\frac{\pi}{4} < x < \pi \end{cases}$



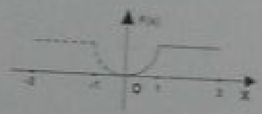
$a_0 = \frac{1}{\pi} \cdot \int_{-\pi/4}^{\pi} x dx = \frac{5}{4}\pi$

$a_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi} \pi \cos nx dx = \frac{1}{n} \sin x \Big|_{-\pi/4}^{\pi} = \frac{1}{n} \sin \frac{n\pi}{4}$

$b_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi} \pi \sin nx dx = -\frac{1}{n} \cos nx \Big|_{-\pi/4}^{\pi} = \frac{1}{n}\left[\cos\frac{n\pi}{4} - (-1)^n\right]$

Ответ: $f(x) = \frac{5}{8}\pi + \sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{n\pi}{4} \cos nx + \frac{1}{n}\left[\cos\frac{n\pi}{4} - (-1)^n\right] \sin nx$

7. $f(x) = \begin{cases} x^2 & 0 < x < 1 \\ 1 & 1 < x < 3 \end{cases}$ по косинусам



$a_0 = \frac{2}{3} \cdot \left(\int_0^1 x^2 dx + \int_1^3 dx \right) = \frac{14}{9}$

$a_n = \frac{2}{3} \cdot \left(\int_0^1 x^2 \cos\frac{n\pi x}{3} dx + \int_1^3 \cos\frac{n\pi x}{3} dx \right) =$

$= \frac{2}{3}\left[\frac{3x^2}{n\pi} \sin\frac{n\pi x}{3} + \frac{18x}{n^2\pi^2} \cos\frac{n\pi x}{3} - \frac{54}{n^3\pi^3} \sin\frac{n\pi x}{3} \Big|_0^1 + \frac{3}{n\pi} \sin\frac{n\pi x}{3} \Big|_1^3 \right] =$

$= \frac{12}{n^2\pi^2}\left[\cos\frac{n\pi}{3} - \frac{3}{n\pi} \sin\frac{n\pi}{3} \right]$

Ответ: $f(x) = \frac{7}{9} + \frac{12}{\pi^2} \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot \left(\cos\frac{n\pi}{3} - \frac{3}{n\pi} \sin\frac{n\pi}{3} \right) \cdot \cos\frac{n\pi x}{3}$